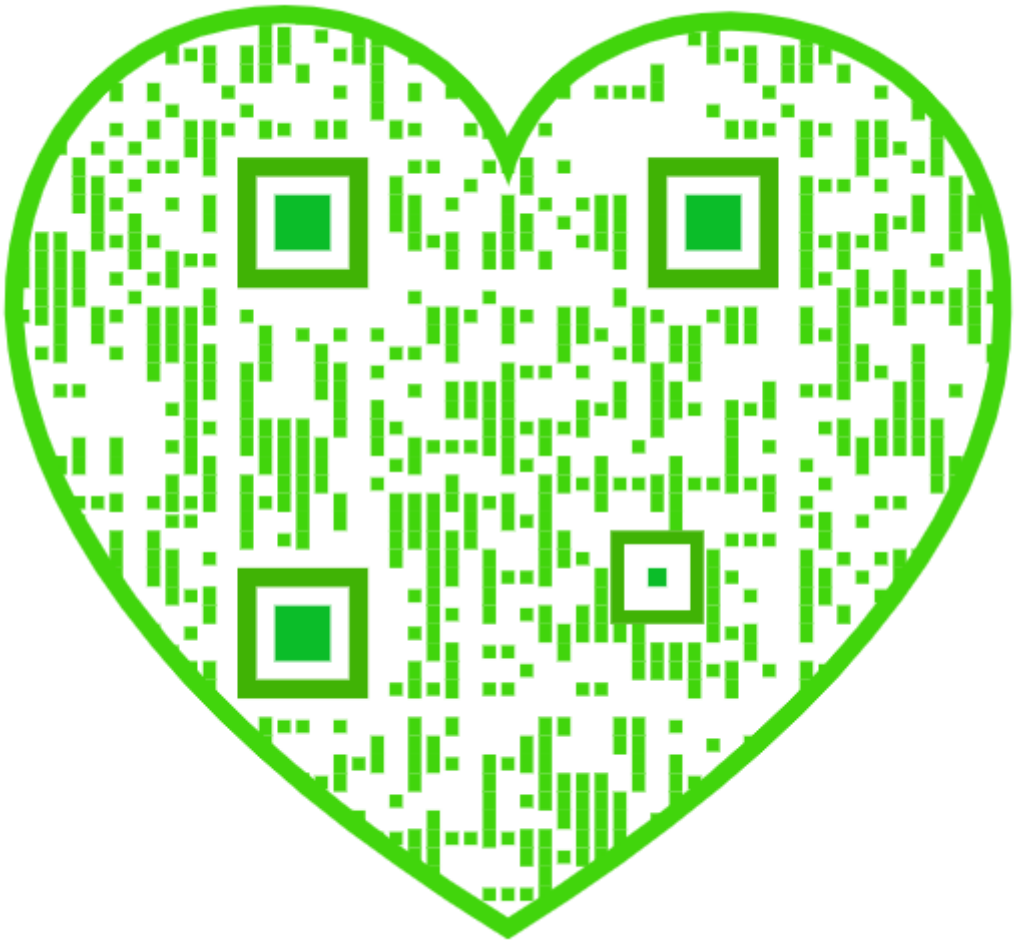

PROGRESSIVE ARITHMETIC

SECOND BOOK

BY

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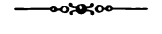

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SECOND PROG. ARITH.

E-P 1

PREFACE

THIS book is designed to cover the work in arithmetic usually given to pupils during the fifth and sixth years of their school course.

It begins with a brief but comprehensive review of the work of the first four years, preparing the pupils to enter upon the study of the new topics with intelligence and ease. Reviews in various forms are, indeed, a feature of the book. The second part, like the first, begins with a thorough review of everything that has preceded, and each part ends with a series of industrial problems designed to apply in the most varied form the principles that have been taught. In addition to these general reviews, each advanced step in any topic is preceded by a brief résumé of the concepts already acquired, so that the pupil may attain and retain a comprehensive grasp of the whole.

The exercises, both oral and written, have been carefully graded, making the advance from one step to the next easy and natural. They are so numerous and varied that the pupils can not fail to become thoroughly familiar with numbers and with the practical processes of computation.

The problems have been framed with the greatest care. They are not numerical puzzles, nor are they based on unreal

conditions. They have been made both rational and practical, and they relate to a wide range of subjects drawn from modern life and industries. The information they embody has been gathered from reliable sources. The prime object of the book, however, is not to convey information on extraneous topics, but to teach the processes of arithmetic in the best way possible.

Great care has been expended on the method of presenting the various principles and on the solutions, with the aim of making them in the highest degree clear, concise, accurate, and practical.

WILLIAM J. MILNE.

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PROGRESSIVE ARITHMETIC

SECOND BOOK

PART I

PRELIMINARY REVIEW

Addition and Subtraction

1. 1. Read $25 + 7 = 32$; $3 + 6 + 8 = 17$.

2. Tell what the sign + indicates; the sign =.

3. When two or more numbers are united into one number by addition, what is the result called?

4. Read $10 - 9 = 1$. What does the sign − indicate?

5. How much greater is 175 than 60? How do you find the answer? What is the answer called?

6. Subtract 486 from 500. Which number is the minuend? Which is the subtrahend? How may the answer be tested?

7. Add 23 to 45. Tell how the units and tens of the sum are obtained.

8. Add 46 to 28. Add 879 to 1668. Explain the process.

9. Subtract 44 from 95; test the result; explain.

10. Subtract 236 from 362, test, and explain.

11. If you sold a person something for 85 cents and received $2 in payment, how should you count out the change?

Add and test:

12. $98 + 7588 + 5009 + 777 + 4275 + 6235 + 4900$.

13. $\$7.59 + \$8167 + \$7526 + \$324.50 + \$.75 + \$7350.50$.

14. $56{,}300 + 4 + 99 + 999 + 9999 + 99{,}999 + 999{,}999 + 99{,}999$.

Subtract from a dollar:

15. 44¢	**17.** 36¢	**19.** 29¢	**21.** 17¢
16. 51¢	**18.** 85¢	**20.** 74¢	**22.** 43¢

Subtract from two dollars; test each result:

23. $1.25	**25.** $1.45	**27.** $0.82	**29.** $1.17
24. $1.50	**26.** $0.12	**28.** $1.64	**30.** $0.85

31. Tell how to count the change out of $2 for each of the amounts mentioned in exercises 23–30.

Multiplication

2. 1. Find the value of $67 + 67 + 67 + 67 + 67 + 67$ by addition; by multiplication.

2. Draw an oblong 4 inches long and 3 inches wide, and divide it into inch squares. Use the oblong to show why 4 times 3 is equal to 3 times 4.

3. Show with dots or marks that 6 times 8 = 8 times 6.

4. Multiply 48 by 23. Point out the multiplicand, the multiplier, and the product. Test your answer.

5. Multiply 347 by 265. Point out the units' partial product; the tens' partial product; the hundreds' partial product. Under what figure of the multiplicand is the right-hand figure of each partial product written?

6. Multiply 274 by 204. Which partial product is not written?

7. Multiply 96 by 2004. When there are 0's in the multiplier, how is the process of multiplication shortened?

8. Multiply 45 by 10; 36 by 100; 725 by 1000.

How are the products found?

How may a whole number (called also *integer*, or *integral number*) be multiplied by 10, 100, or 1000?

Division

3. 1. Divide 1000 by 77. Point out the dividend, the divisor, the quotient, and the remainder.

2. How may the answer in exercise 1 be tested?

3. Read the following, using the words "divided by":

$$66 \div 3 = 22 \qquad 5\overline{)75} \qquad \frac{42}{21} = 2$$

Point out the dividend, the divisor, and the quotient in each case. Test each result.

4. Tell how $\frac{1}{5}$ of 365 days is found.

Indicate the process, using the signs $\div$ and $=$.

5. How many times does 1000 bushels contain 8 bushels?

Indicate the process in three different ways.

6. With what divisors is short division used?

7. Divide 8000 by 20; 8411 by 40.

How is an integer divided by another integer ending in 0?

What is done in such a case if the dividend does not end in 0?

8. Without writing any figures, divide 5280 by 10; by 100; by 1000. Tell how you perform these divisions.

9. Divide 3843 by 200.

Write the result as quotient and remainder.

Write it in the form of a whole number and a fraction, or in the form of a *mixed number.*

10. If 4 yards of silk cost \$9, how many dollars will 1 yard cost? Test your answer.

11. I have \$9. How many 4-dollar hats can I buy and how much money shall I have left?

12. Edward divided 72 by 13, obtaining 5 for a quotient and 6 for a remainder. Test his answer.

13. Divide 16,854 by 36.

Use your work to illustrate the answers to the following questions:

14. If the quotient figure should be taken too large, how would the error be discovered? if it should be taken too small?

15. When the proper quotient figure is taken, how is the fact shown by the remainder?

16. Divide, and test each result by multiplication:

$2\overline{)608}$ $5\overline{)2005}$ $12\overline{)1296}$ $12\overline{)6108}$ $8\overline{)241648}$

When any remainder with the next figure of the dividend does not contain the divisor, what figure is written in the quotient?

Common and Decimal Fractions

4. **1.** Read the following fractions and name the numerator and the denominator of each:

$\frac{3}{4}$ $\frac{5}{6}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{5}{4}$ $\frac{7}{16}$ $\frac{10}{10}$ $\frac{9}{2}$

2. What does each fraction in exercise 1 denote?

3. Which fractions are *proper* and which are *improper?*

4. Add $\frac{3}{4}$ and $\frac{5}{8}$. Explain your work.

5. Which is the greater, $\frac{1}{2}$ or $\frac{3}{4}$? How do you know? Illustrate with a drawing.

6. If two fractions have different denominators, what must be done before the fractions can be united into a single fraction by addition or subtraction? Illustrate with examples of your own.

7. Which of the following fractions are in their lowest terms?

$\frac{5}{7}$ $\frac{8}{16}$ $\frac{14}{32}$ $\frac{9}{6}$ $\frac{10}{12}$ $\frac{8}{15}$ $\frac{9}{24}$ $\frac{32}{18}$

Express them all in their lowest terms.

8. Add $18\frac{2}{3}$ to $25\frac{3}{4}$. Explain your work.

9. Subtract $16\frac{2}{8}$ from 100. Explain.

10. Reduce $4\frac{5}{8}$ to an improper fraction. Explain.

11. Reduce $\frac{33}{10}$ to a mixed number. Explain.

12. Find $\frac{4}{5}$ of 73,245.

Tell how to find $\frac{4}{5}$ of any number; $\frac{3}{8}$ of any number.

13. Multiply 5280 by $2\frac{5}{11}$.

Tell how to multiply an integer by a mixed number.

14. A decimal fraction is one that expresses tenths, or hundredths, or thousandths, etc. All other fractions are common fractions.

Which of the following are decimal fractions and which are common fractions?

$\frac{1}{2}$ .5 $\frac{5}{10}$ $\frac{3}{5}$ .72 $\frac{9}{100}$ .325 $\frac{7}{8}$

15. Reduce to decimal fractions:

$\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{1}{20}$ $\frac{3}{4}$ $\frac{3}{5}$

16. Reduce to common fractions:

.6 .25 .75 .16 .8 .40 .64 .125

17. Add $\frac{1}{2}$ and $\frac{3}{10}$; $\frac{3}{4}$ and .2; $\frac{1}{5}$ and .25.

18. How much greater is $\frac{3}{8}$ than $\frac{3}{10}$? $\frac{1}{2}$ than .45? $\frac{3}{4}$ than .6? .8 than .55? $\frac{4}{10}$ than $\frac{7}{25}$?

Measurements and Comparisons

5. 1. What measures are used in buying and selling milk? peanuts? grain? hay? coal? kerosene?

2. How are long distances measured? short distances?

3. What measures are used in measuring land?

4. Give the table of measures of time.

5. Name the months and tell the number of days in each.

6. Tell why a square foot contains 144 square inches. How is the number of cubic inches in a cubic foot found?

7. Compare 1 yard with 1 foot in two ways:

1 yd. is —— ft. longer than 1 ft.;
1 yd. is —— times 1 ft.

8. Compare 6 with 8 in two ways:

6 is —— less than 8; 6 is —— of 8.

Compare in two ways:

9. 1 ft. with 2 in.
10. 1 gal. with 2 qt.
11. 1 gal. with 1 pt.
12. 1 lb. with 4 oz.
13. 2 bu. with 1 pk.
14. 1 hr. with 12 min.
15. 24 with 6.
16. 6 with 24.
17. 25 with 100.
18. 75 with 100.
19. 60 with 100.
20. 120 with 100.

Which is the greater and how much:

21. 6×8 or 7×7?
22. 7×9 or 8×8?
23. 3×15 or 4×12?
24. 4×20 or 3×25?
25. $81 \div 9$ or $42 \div 7$?
26. $45 \div 3$ or $32 \div 2$?
27. $\frac{1}{3}$ of 66 or $\frac{2}{3}$ of 36?
28. $\frac{1}{2}$ of 100 or $\frac{2}{3}$ of 60?

Miscellaneous Problems

SUGGESTION. — When the problems are related to one another, the answer to each should be kept until the series is completed.

6. 1. Edith invited 11 friends of her own age to a party to celebrate her eleventh birthday. What were the combined ages of her guests?

2. She bought invitation cards with envelopes at 2¢ each, and used 2-cent stamps. Find the cost of sending invitations.

3. Edith bought 2 loaves of bread @ 10¢ and cut from

each 16 slices. Each slice made one sandwich. How much did the bread for each sandwich cost?

4. A chicken costing 64¢ and $\frac{3}{8}$ lb. butter @ 32¢ were used for the sandwiches. What was the total cost of each sandwich?

5. Edith bought 4 dozen fancy cakes at 6 for 5¢ and a birthday cake for $1.25. Find her change out of $5.

6. She made 7 quarts of lemonade. How many glasses, $\frac{1}{2}$ pt. to a glass, did this allow for each child, and how many over?

7. One lemon was used for every pint, and one was left over. Find the cost of the lemons purchased, at 3 for 5¢.

8. If a quart of ice cream will serve 8 children, how many quarts were needed, allowing enough for 8 second helpings?

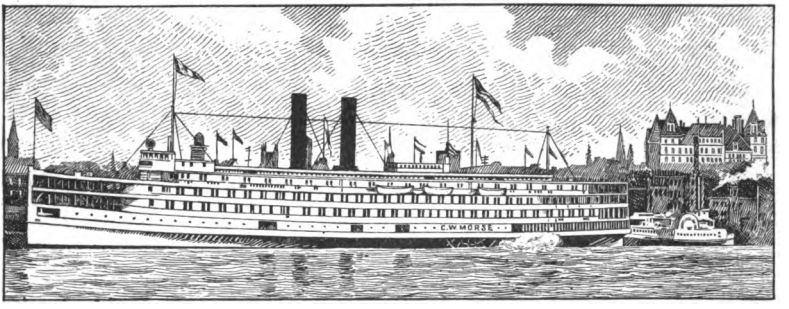
9. At 40¢ a quart, what was the cost of the $2\frac{1}{2}$ quarts of ice cream? How much did all the refreshments cost?

10. The children played at tossing rings on pegs. A ring on the center peg counted 20, but on any corner peg only 5. What was Albert's score, if he threw the ring 6 times on the center peg, and 15 times on the corner pegs?

11. Marjorie arrived at 3:15 P.M. and left at 5:42 P.M. How long was she at the party?

12. Robert and his uncle went from Albany to New York on the night boat. Robert bought a half-fare ticket, 75¢, and his uncle a full-fare ticket. They bought a stateroom ticket for $2. How much did they pay for tickets?

13. Other expenses of the trip were: dinner, $2.25; magazines, 40¢; Panorama of the Hudson, $1; 15 souvenir post cards at 3 for 10¢; morning paper, 5¢. What was the sum of these expenses?



14. Find the whole cost of the trip.

15. The purser told Robert's uncle that there were 1600 passengers on board that night; $\frac{3}{4}$ of them paid full fare ($1.50) and $\frac{1}{5}$ paid half fare. The rest were steerage passengers and paid $1 each. Find the receipts for fares.

16. The staterooms were all occupied. The boat had 110 @ $1, 200 @ $2, 30 @ $3, 2 @ $4, 10 @ $5, and 4 @ $6. What were the receipts for staterooms?

17. Find the receipts from 182 cabin berths at 50¢ each.

18. During the trip 396 meals were served at an average price of $1.25 each. Find the receipts of the dining room.

19. It costs 5¢ to check an article, and the following articles were checked: 138 suit cases, 97 overcoats, and 48 hand satchels. What were the receipts of the check room?

20. The steamer carried freight that night as follows :

Class	Weight	Rate per 100 lb.
First	29¾ tons	20¢
Second	19½ "	15¢
Third	51¼ "	12¢
Fourth	127½ "	11¢
Fifth	283 "	10¢
Sixth	472 "	8¢

What were the receipts for freight?

21. Find the total receipts from fares, staterooms, cabin berths, dining room, check room, and freight.

22. If the average salary of each man employed was $580 for a season of 290 trips, what was the amount paid the entire crew of 135 men for one trip?

23. If the receipts of the dining room amounted to $495 for the trip and the cost of the supplies was ⅖ of that amount, how much did the supplies cost?

24. The steamer uses 35 tons of pea coal a trip. Find the cost at $4.90 per ton.

25. If 9 gallons of lubricating oil are used in the engine room on one trip, find the cost at 67¢ per gallon.

26. Find the cost of the cotton waste used, 7 pounds at 9½¢ a pound.

27. It took 45 men 9 hours to unload and reload the freight. Find the cost, if each man received 30¢ per hour.

28. Find the total cost for wages of crew, dining room supplies, coal, lubricating oil, cotton waste, and handling freight.

29. The distance from Albany to New York is 143 miles. If the steamer makes the trip in 10 hours, find its rate per hour.

30. If the same steamer sails at the rate of 13 miles per hour in returning to Albany from New York, how much longer does it take for the return trip?

NOTATION AND NUMERATION

7. The people of Europe first learned the use of the figures **1**, **2**, **3**, **4**, **5**, **6**, **7**, **8**, **9**, and **0** from the Arabs.

These ten figures are therefore called the **Arabic numerals.**

8. The method of representing numbers by means of the ten Arabic numerals is called the **Arabic notation.**

9. Any method of naming or reading numbers is called **numeration.**

10. **1.** Count by ones to 9; by tens to 90; by hundreds to 900; by thousands to 9000.

2. Ones are called **units of the first order** or simply **units**; tens are called **units of the second order**; hundreds are called **units of the third order**; and so on.

11. In the Arabic notation:

(1) *The greatest number of units of any order is nine.*

(2) *Therefore ten units of any order are written as one unit of the next higher order.*

The Arabic notation, being based on the number ten, is often called the **decimal notation**, from the Latin word *decem*, meaning ten.

12. **1.** Read 8, 28, 46, 104, 208, 500, 987, 1000.

If you have read the numbers correctly, you will have illustrated these three facts about reading numbers:

(1) *The figure* 0 *is not read.*

(2) *The word "and" is not used in reading integers.*

(3) *Tens and units and also hundreds, tens, and units are read together, as units.*

Thus 987 is not read 9 hundreds, 8 tens, 7 units.

It may be read "nine hundred eighty-seven units," but the word "units" is usually omitted.

2. Hundreds, tens, and units constitute **units' period.**

3. When we reach a thousand, we begin to name the thousands in order as we do the units, thus:

1 thousand, 2 thousand, etc., up to 999 thousand.

Thousands, ten-thousands, and hundred-thousands constitute **thousands' period.**

4. Write with figures 654 thousand 528 (units).

Read 362,475; 502,308; 750,000.

5. Just as 1000 units make 1 thousand, so 1000 thousands make 1 **million** (1,000,000), and 1000 millions make 1 **billion** (1,000,000,000).

TABLE

BILLIONS' PERIOD				MILLIONS' PERIOD				THOUSANDS' PERIOD				UNITS' PERIOD		
Hundred-billions	Ten-billions	Billions	,	Hundred-millions	Ten-millions	Millions	,	Hundred-thousands	Ten-thousands	Thousands	,	Hundreds	Tens	Units
3	6	5	,	4	8	2	,	7	2	5	,	9	3	8
		3	,	8	3	6	,	0	3	9	,	4	5	7
		72	,	0	0	0	,	0	2	4	,	6	0	6
8	0	4	,	2	0	0	,	3	4	1	,	0	0	0

6. The first number in the table is read:

"365 *billion*, 482 *million*, 725 *thousand*, 938."

Read the other numbers in the table.

13. To read a large number:

First, beginning at the right, separate the figures by commas into periods of three figures each.

Next, beginning at the left, read each period as if it stood alone, adding its name.

The highest period may contain less than three figures.

14. 1. How many tens are there in 1 hundred? units in 1 ten? tenths in 1 unit? hundredths in 1 tenth? thousandths in 1 hundredth?

2. .5 denotes 5 tenths and is so read; .56 denotes 5 tenths and 6 hundredths, and is read "56 hundredths"; .567 is read "567 thousandths."

3. Read, and tell what each figure denotes:

.8	.62	.37	.80	.094	.140	.005
.7	.60	.07	.84	.095	.014	.301
.9	.78	.48	.92	.105	.404	.070
.6	.51	.34	.09	.125	.382	.800

15. *The Arabic notation is used to express integers and decimals. Ten units of any order, integral or decimal, make one unit of the next higher order.*

A dot called the **decimal point** is placed before tenths' figure.

Tenths occupy the *first* decimal place, *hundredths* the *second*, *thousandths* the *third*, and so on.

16. 1. Read the following mixed numbers:

$$7\tfrac{5}{10},\ 7.5,\ 4\tfrac{7}{100},\ 4.07,\ 5\tfrac{168}{1000},\ 5.168.$$

A mixed number that is expressed as an integer and a decimal is called a **mixed decimal.**

In reading mixed decimals "and" is used between the name of the integer and that of the fraction, but not elsewhere.

2. Read, and tell what each figure denotes:

5.3	6.32	28.41	1.125	36.404
8.7	4.75	50.67	3.146	80.225
6.5	8.06	32.04	8.073	37.670
3.9	5.88	90.02	6.008	20.006

EXERCISES

17. Read:

1. 425,360.2
2. 632,587.09
3. 1,789.346
4. 426,502.609
5. 2,000.2
6. 55,000.55
7. 1,001,000.001
8. 105,000,005
9. 6,275,200.003
10. 49,482,376.735
11. 928,346,002
12. 3,003,030,300
13. 72,485,364.6
14. 864,375,000

Copy, writing units of the same order in the same column, point off into periods, and read:

15. 4.9, 17.02, 195.005, 3850, 6500000
16. .75, 508.35, 626271, 6424365, 730.402
17. 4.52, 578 96, 24329.05, 1487625.9
18. 3.003, 860.7, 24867.3, 49278564
19. 5781.2, 1.008, 320.75, 48356033.8
20. 73348, 425637, 42235679, 365289073

WRITTEN EXERCISES

18. 1. Write the names of the orders of integers and decimals from hundred-billions down to thousandths.

Express in the Arabic notation:

2. 4 hundreds 6 tens 3 units 5 tenths.
3. 9 tens 9 units 9 tenths 9 thousandths.
4. 3 billions 3 hundred-thousands 3 tens.
5. 7 ten-millions 8 thousands 6 units 8 tenths.
6. Write the numbers that are one greater than:

1000 1199 6229 9089 6011 9999

7. Write the numbers that are 1 less than:

100 500 1621 2000 11,000 100,000

Express in the Arabic notation:

8. 107 million, 875 thousand, 596 (units).

9. 220 billion, 625 million, 340 (units).

10. 4 billion, 362 million, 256 thousand.

11. 400 million, 870, and 2 tenths.

12. 5 billion, 5 million, 5 thousand, 5 hundred.

13. 6 thousand, 325, and 46 hundredths.

14. 1000 thousand, 50, and 4 tenths.

15. 1000 million, 325 thousand, 411.

16. 625 thousand, 75, and 12 hundredths.

17. 48 thousand, and 9 thousandths.

18. 7 million, 7 thousand, 7.

19. 100 million, 1 thousand, 1.

20. Two hundred two thousand, sixty.

21. Sixteen million, forty-five thousand.

22. One hundred million, seven hundred fifty.

23. Forty-three million, seventy-six thousand.

24. Ninety million, ninety thousand, ninety-one.

25. Nine hundred ninety-nine thousand, ninety-nine, and nine hundredths.

26. Sixty-two million, nineteen thousand, seven hundred, and seven hundredths.

27. Seventeen hundred three.

28. Fourteen hundred million, five hundred thousand two hundred ninety-seven.

Write in words:

29. 19, 90, 99, 101, 320, 514, 711, 7.11, 6.009

30. 705, 700.5, 700.705, 2000.2, 600.003, .603

31. 33,485,620.033, 650,000,001.010, 428,000.866

United States Money

19. 1. What part of a dollar is 1 dime?

2. What part of a dime is 1 cent? What part of a dollar is 1 cent?

3. What part of a dollar is $\frac{1}{10}$ of a cent?

4. A thousandth of a dollar is called a **mill.**

5. \$.01 is read "1 cent"; \$.001 is read "1 mill"; \$.156 is read "15 cents 6 mills"; \$2.308 is read "2 dollars, 30 cents, 8 mills."

25.4¢ is read "25 and 4 tenths cents."

6. Compare with $\frac{1}{2}$ cent: 5 mills; 4 mills; 6 mills.

7. Mills are not coined. If a man owes \$2.123, he usually must pay \$2.13.

EXERCISES

20. Read:

1. \$.11, \$3.25, \$6.75, \$192.20, \$750.05

2. \$3.225, \$48.368, \$17.213, \$277.312, \$.04

3. \$25.826, \$492.375, \$963.527, \$.875, \$.006

4. Write in words: \$2.25, \$3.05, \$.12$\frac{1}{2}$, \$.375, \$.108

5. Write with figures and the dollar sign:

Eighty-two dollars, ten cents, two mills; ten dollars, nine cents, five mills; one thousand dollars, one cent, eight mills; fifteen thousand fifteen dollars, fifteen cents.

6. Read, then write as cents, using the sign ¢:

\$4.62	\$3.33	\$3.333	\$10.01	\$25.034
\$0.09	\$1.125	\$16.08	\$100.05	\$42.375

7. Read, then write as dollars and cents:

365¢	1100¢	1280¢	55410¢	177.2¢
1908¢	2807¢	1609.5¢	20009¢	184.9¢

ADDITION

EXERCISES

21. Add, giving results quickly:

Add the tens of one number to the whole of the other, and then add the units, thus in exercise 1, "57, 87, 93."

1.	57	28	46	39	22	57	34	62
	36	63	42	54	69	27	46	29
2.	93	75	49	83	26	78	69	34
	38	46	64	45	57	39	63	75
3.	77	84	56	43	64	85	76	99
	47	96	79	58	87	66	98	88
4.	28	65	49	37	78	25	57	18
	39	75	26	84	36	49	64	86

Add rapidly:

5.	\$7	**6.**	9¢	**7.**	8 lb.	**8.**	3 bu.	**9.**	5 rd.
	8		6¢		5 lb.		9 bu.		8 rd.
	5		8¢		6 lb.		8 bu.		7 rd.
10.	\$9	**11.**	8¢	**12.**	4 in.	**13.**	7 pt.	**14.**	9 gal.
	5		7¢		9 in.		3 pt.		7 gal.
	6		5¢		9 in.		6 pt.		8 gal.

22. The process of finding a number that is equal to two or more given numbers is called **addition.**

23. The numbers added are called **addends**, and the result found by adding is called the **sum**.

24. The numbers \$2 and \$5 are *like numbers;* so also are 6 qt. and 8 qt.; also 3 and 10.

Give other like numbers.

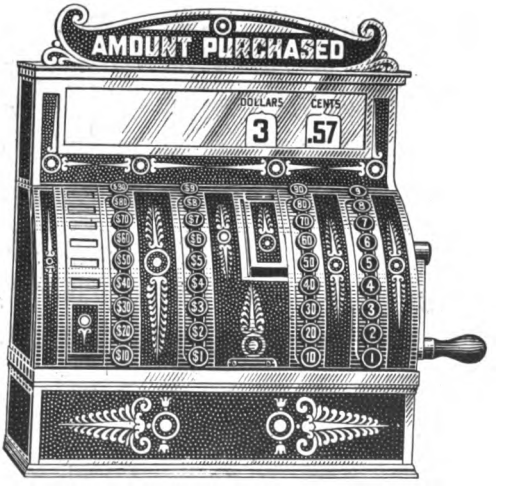
25. 7 sheep and 9 cows are *unlike numbers*, but 7 animals and 9 animals are *like numbers*.

26. Add \$2 and \$5; 6 qt. and 8 qt. Can you add 7 sheep and 9 cows? 7 animals and 9 animals?

Only like numbers can be added.

EXERCISES IN MAKING CHANGE

27. The picture shows a cash register. When a purchase amounting to, say, \$3.57 is made, the clerk presses the keys marked \$3, 50, and 7. This registers the purchase on a slip of paper within, pushes up cards that show the amount of the purchase, and opens the cash drawer.



Suppose the purchaser has given the clerk \$5. The clerk puts the money into the drawer and may take out the change thus: three 1-cent pieces, a dime, a 5-cent piece, a quarter, and a dollar. He then hands the change to the purchaser in the same order, saying as he does so: "Three fifty-seven, sixty, seventy, seventy-five, four dollars, five dollars."

Following are the amounts of some purchases and the sum of money paid the clerk in each case:

Tell what you would do if you were clerk — the keys of the cash register you would press, the pieces of money you would take out of the drawer, and what you would say to the purchaser as you handed him the change.

	Purchase	Money Paid		Purchase	Money Paid
1.	$1.38	$2.00	13.	$3.69	$5.00
2.	2.13	3.00	14.	1.47	2.00
3.	.42	1.00	15.	2.24	4.00
4.	1.76	5.00	16.	6.45	7.00
5.	2.84	4.00	17.	8.72	10.00
6.	.77	2.00	18.	7.66	9.00
7.	4.09	5.00	19.	5.23	10.00
8.	3.61	5.00	20.	3.17	10.00
9.	1.15	1.50	21.	7.14	8.00
10.	2.03	4.00	22.	8.21	10.00
11.	3.11	3.50	23.	7.25	20.00
12.	2.44	5.00	24.	9.48	20.00

EXERCISES

28. Add rapidly, both columns at once, thus in exercise 1, adding upward, "11, 20, 30, 36, 44, 54, 65":

1.	2.	3.	4.	5.
11	10	11	10	9
10	11	10	8	10
8	5	11	10	8
6	10	9	11	11
10	10	3	4	6
9	7	11	11	10
11	8	6	10	3

Add rapidly, both columns at once:

6.	7.	8.	9.	10.
10	11	9	10	11
9	6	10	8	10
8	10	11	9	9
11	8	7	11	6
10	5	6	3	11
7	10	10	10	5
10	9	4	7	10
5	11	6	10	6

11.	12.	13.	14.	15.
9	10	11	6	11
11	4	7	5	6
6	11	8	11	10
10	9	10	10	11
11	6	4	8	10
7	11	11	10	7
11	10	5	11	6
8	9	10	9	9
10	8	6	10	11

16.

3	
4	10
3	
5	
6	11
9	
4	
3	7
6	
4	10
8	
55	

Learn to catch the sums of groups of numbers and add them, reading up the column rapidly. Thus, in the column in the margin read, "8, 18, 25, 34, 45, 55."

Where possible, use groups of 10 or 11, as these sums are more easily added, but combine into any convenient groups. It is best not to pass over numbers in order to group by 10's, for there is danger of neglecting numbers once passed over.

By practice you will be able to use groups that are larger and larger, but for the present they should not be over 11.

Practice until you can add all the following columns in less than $1\frac{1}{2}$ minutes.

17.	18.	19.	20.	21.	22.	23.
5	3	8	4	2	7	5
3	6	3	7	8	6	7
2	7	5	8	3	3	3
6	4	4	6	7	9	4
7	4	6	9	8	4	7
3	6	9	2	3	7	9
2	9	5	5	2	8	2
9	1	2	5	4	3	6
7	3	7	2	4	4	4
5	2	8	3	3	5	9
5	5	2	5	5	5	8
8	7	6	8	6	4	2
2	3	4	2	7	6	3
9	5	5	6	3	9	7
7	6	5	4	5	6	8

WRITTEN EXERCISES

29. In adding numbers of two or more columns, the sum of each column should be retained in some way so that in case of error or interruption while adding any column it will not be necessary to go over all the previous columns.

A convenient method often used by business men is illustrated in the margin, the sums being written on a separate piece of paper. The sums of the columns, so written, give the required sum.

1.
$4.39
3.64
5.98
7.75
6.83
$28.59

29
33
25

CAUTION. — When this method of keeping the sums of the columns is used, care should be taken not to "carry" from one column to the next.

Add by grouping, and test each result by adding in the opposite direction:

Exercises 2–6 have been added and tested in less than 5 minutes. Can you do better?

2.	3.	4.	5.	6.
$3.46	2648	$46.28	97.38	27.456
8.75	3462	17.32	4.75	46.734
.63	5732	54.59	56.86	8.463
8.96	1897	9.81	37.91	2.057
2.47	314	2.27	24.29	43.591
.64	9285	47.96	8.63	38.762
4.27	8625	63.84	.58	76.357
6.93	3974	15.38	49.63	14.205
5.26	7286	34.02	12.74	7.836
3.72	4738	8.46	56.37	.074
2.48	3473	3.25	82.46	8.698

Practice until you can add and test exercises 7–11 in less than 8 minutes:

7.	8.	9.	10.	11.
28,749	$537.24	$46.32	932.468	684,231
36,451	394.66	24.724	825.510	598,763
54,937	716.84	48.066	253.132	3,821,475
80,783	29.35	79.827	984.749	5,279,635
25,436	82.75	12.283	229.861	2,634,798
93,617	894.33	95.75	671.154	275,824
74,262	323.04	69.31	528.187	743,186
43,231	462.53	24.649	163.728	194,089
54,768	659.21	13.371	429.351	523,716
56,342	51.36	83.18	740.286	8,576,371
79,506	978.03	39.724	967.345	425,843
86,789	792.14	59.869	479.683	176,928
34,321	747.96	54.627	937.427	934,182

WRITTEN EXERCISES

30. 1. A dealer in poultry sold 4800 eggs in January, 2160 in February, and 6708 in March. How many eggs did he sell in the three months?

2. Add: forty-nine thousand eight hundred seventeen, twenty, thirty thousand five hundred forty-five, sixteen thousand eight hundred ninety, and six thousand twelve.

3. What is the sum of 2685 dollars 32 cents, 476 dollars 9 cents, 1020 dollars 10 cents, and 946 dollars 87 cents?

4. Westport is 127.78 mi. north of Albany and 63.13 mi. south of Rouses Point, on a railroad joining these two places. How far is it from Albany to Rouses Point?

5. A farmer sold his potatoes for $1085, wheat for $248.25, rye for $176.45, and hay for $369.50. How much did he receive for all?

6. A builder bought a lot for $584. He built a house upon it costing $4362 and a barn for $974.50. He paid $293.25 for walks, grading, and setting out trees. For how much must he sell the property to gain $775?

7. How many rods of fence are needed to inclose a field whose sides are 27.5 rods, 84.125 rods, 46.08 rods, 62.504 rods, and 18.32 rods long, respectively?

8. In 1904, the popular vote for Roosevelt was 7,624,489; for Parker, 5,082,754; for Debs, 402,286; for Swallow, 258,787; for Watson, 117,935; and for Corrigan, 32,088. What was the total vote for all candidates?

9. One year the number of foreigners entering the United States was as follows: Germans, 46,380; Italians, 193,246; Russians, 145,141; Irish, 36,142; English, 38,626; Japanese, 14,264; all other nationalities, 339,071. What was the total immigration that year?

10. Add: seven and eighty-one hundredths, eighteen and two hundred forty-six thousandths, forty-five and twelve thousandths, two hundred seventy-four and seven hundredths, and sixty-eight and nine thousandths.

11. How many pounds of cotton were there in a shipment of 12 bales, weighing respectively, 504, 488, 496, 512, 519, 497, 525, 508, 499, 514, 487, and 513 pounds?

12. In a certain year the grain production of the United States was: oats, 784,094,199 bu.; buckwheat, 14,243,644 bu.; barley, 131,861,391 bu.; Indian corn, 2,244,176,925 bu.; rye, 29,363,016 bu.; wheat, 637,821,835 bu. How many bushels of grain did the country produce that year?

SUBTRACTION

EXERCISES

31. Subtract, giving results quickly:

Subtract the tens of one number from the whole of the other and then subtract the units, thus in exercise 1, "75, 35, 26."

1.	75	52	61	46	88	72	55	99
	49	36	29	18	25	37	29	63
2.	48	67	81	56	44	91	73	85
	19	42	33	27	16	59	25	68
3.	\$83	78¢	56 in.	92 oz.	64 mo.	75 doz.		
	38	53¢	17 in.	45 oz.	28 mo.	46 doz.		
4.	\$1.72	\$1.10	\$2.95	\$1.46	\$2.67	\$1.13		
	.45	.76	.54	.63	.49	.85		

32. The process of finding how much greater one number is than another, or of taking part of a number from it and finding how many are left, is called **subtraction.**

33. The greater number is called the **minuend,** the smaller one is called the **subtrahend.**

34. The result found by subtracting is called the **difference** or **remainder.**

35. *The minuend and subtrahend must be like numbers.*

EXERCISES

36. 1. Mary's geography cost 75 ¢ and her reader 39 ¢. How much more did her geography cost than her reader?

2. Mr. Morton had $85 and spent $46 of it for a rug. How much money had he left?

3. If 74 is the minuend and 35 is the subtrahend, what is the remainder?

4. There were 34 collie dogs at a dog show. All but 18 of them belonged to Mr. Cory. How many did Mr. Cory exhibit?

5. A man earns $112 a month and spends $88 in that time. How much does he save per month?

6. How much change from a dollar bill will a boy receive who buys a ball for 69 cents?

7. Mr. Barnes has 96 sheep in three lots. In one lot there are 25 and in another 34. How many are in the third lot?

8. Find the sum of 48 and 36, and subtract it from 114. What is the remainder?

9. By selling a horse for $125 I lost $32. If I had sold him for $175, should I have gained or lost and how much?

10. Mrs. James bought a pail for 25 ¢ and a dozen pans for 84 ¢. How much change did she receive out of $2?

In each of the following, find, by subtracting the amount of the purchase from the money paid, how much change should be given.

	Purchase	Money Paid		Purchase	Money Paid
11.	$1.45	$2.00	21.	$2.33	$5.00
12.	.75	1.00	22.	1.12	1.50
13.	2.25	3.00	23.	2.61	2.75
14.	3.50	5.00	24.	3.35	4.00
15.	.49	2.00	25.	2.47	5.00
16.	.85	2.00	26.	.82	5.00
17.	4.68	5.00	27.	2.09	3.00
18.	2.94	5.00	28.	6.55	10.00
19.	3.41	4.00	29.	2.79	10.00
20.	1.28	2.00	30.	8.31	10.00

If more drill of this kind is desired, do the exercises on page 24 in the same way.

WRITTEN EXERCISES

37. Subtract, and test the result by adding it to the subtrahend; the sum should be equal to the minuend.

Exercises 1–15 have been done correctly in 4 minutes. Practice until you can do better.

1. $7.21	2. 5,836	3. $46.25	4. 59.63	5. 48,794
2.98	2,947	13.81	27.31	16,859
6. $8.46	7. 7,029	8. $63.40	9. 95.68	10. 50,623
3.77	4,634	28.92	14.09	34,748
11. $9.03	12. 9,003	13. 85.00	14. 82.05	15. 70,060
5.89	2,871	37.24	54.67	56,789

Subtract and test:

Practice until you can do the work of this page in less than 13 minutes.

16. \$387.43 179.85	17. 476.73 298.94	18. 876,421 668,376	19. 7,894,725 5,698,947
20. \$502.61 346.29	21. 624.91 468.36	22. 462,385 183,498	23. 6,643,489 4,958,397
24. \$750.30 267.41	25. 900.00 254.62	26. 780,040 562,735	27. 8,036,370 3,276,593
28. \$645.00 359.26	29. 734.14 469.27	30. 948,000 269,123	31. 56,914.32 26,347.84
32. \$86.418 27.305	33. 53.625 34.579	34. 743.981 365.492	35. 43,761.26 14,698.77
36. \$54.363 19.875	37. 89.684 27.595	38. 540.263 274.325	39. 74,326.08 35,368.49
40. \$75.285 34.968	41. 60.805 13.748	42. 673.499 586.944	43. 6,234.521 2,578.936
44. \$92.740 58.475	45. 70.004 34.052	46. 932.068 784.679	47. 8,035.040 4,746.806
48. \$40.000 22.743	49. 81.206 45.987	50. 800.000 245.678	51. 9,748.072 5,879.244

WRITTEN EXERCISES

38. **1.** From the sum of 5391 and 2645 subtract the difference.

2. George Washington was born in the year 1732 and died in 1799. How old was he when he died?

3. America was discovered by Columbus in the year 1492. How many years have passed since that event?

4. Mr. Burton bought a 1000-mile railroad book to use while on his vacation. When he returned home, he had 479 mile tickets left. How many tickets had he used?

5. A man bought a lot for $978 and built a skating rink on it at a cost of $5345. He then sold the whole property for $8150. How much did he gain?

6. Subtract ten dollars ten cents from ten thousand dollars.

7. A merchant bought some goods for $1637.45 and sold them for $1964.94. How much did he gain?

8. If 13,362 school children are enrolled in a certain city and 6693 of them are boys, how many girls are enrolled?

9. There were 207,977 miles of railway in the United States in a certain year, and 213,904 miles the next year. How much was the gain in one year?

10. The sum of two numbers is 3,042,801, and one of them is 2,400,037. What is the other number?

11. From thirty-four and two hundred forty-five thousandths subtract eighteen and nine hundredths.

12. If it takes a letter 154.6 hours to go from New York to London, and 180.6 hours from New York to Paris, how much more quickly will one go to London than to Paris?

13. Mr. Green had a farm of 342.75 acres. He sold from it at different times 37.5 acres, 126.25 acres, and 88.5 acres. How many acres did he have left?

MULTIPLICATION

39. 1. Count by 6's to 12 times 6. Give the table of 6's.

2. Count by 7's to 12 times 7. Give the table.

3. By counting in a similar way, obtain in order the tables of 8's, 9's, 10's, 11's, and 12's.

4. If you do not already know the multiplication tables perfectly, *commit them to memory now.*

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

In this table the 1's are written in the first row, the 2's in the second row, the 3's in the third row, and so on. To find the value of 7 times 8, for example, find 8 in the first column, thus locating the eighth row, in which all the 8's are written. 7 times 8, or 56, will be found in this row in the seventh column, or under the figure 7 in the top row.

DRILL EXERCISES

40. Tell products instantly:

5×6	6×4	8×3	2×5	6×9
3×7	9×3	6×6	8×2	4×4
9×5	3×6	8×5	7×3	8×6
8×7	7×8	9×7	2×9	2×7
3×9	5×7	9×2	9×4	11×2
8×4	7×6	7×4	7×5	11×5

10	11	12	10	11	2	8	2
7	3	2	8	4	12	10	11
11	12	7	11	3	7	11	4
6	5	11	5	12	12	8	12
9	8	12	9	12	11	8	6
7	4	9	5	7	10	12	12
8	9	10	12	9	7	9	11
9	10	12	11	8	6	4	9
6	12	8	12	5	6	9	8
9	8	11	10	12	11	11	5
7	11	10	8	11	10	12	10
7	11	10	8	7	9	6	11
9	7	8	9	12	11	9	12
9	10	6	12	4	12	7	12

41. 1. Find the sum of two 32's; of three 32's; of four 32's; of ten 32's.

2. Tell a short way to find the sum of several 32's.

42. A short process of finding the sum of equal numbers, or the process of taking one number as many times as there are units in another, is called **multiplication.**

43. The number taken or multiplied is called the **multiplicand.**

44. The number that shows how many times the multiplicand is taken is called the **multiplier.**

45. The result obtained by multiplying is called the **product.**

46. 1. How many dollars are 4 times \$20? How many trees are 4 times 20 trees? How many miles are 4 times 20 miles?

2. When 20 things of any kind are multiplied by 4, how many things are there in the product? What kind of things compose the product?

47. A number used without reference to any particular thing is called an **abstract number.**

2, 4, 20, etc., are abstract numbers.

48. A number used in connection with some particular thing is called a **concrete number.**

20 is a concrete number when it refers to dollars, or trees, or miles, etc.

49. Point out abstract and concrete numbers:

15 days	15 years	15	\$12	12¢	12
6	6	6	5	5	5
90 days	90 years	90	\$60	60¢	60

50. *The multiplier must be regarded as abstract. The product is like the multiplicand.*

51. **1.** Compare 20 times 2 with 2 times 20; 12 times 4 with 4 times 12.

In finding the product of two abstract numbers, either may be taken for the multiplicand.

2. 5×11 gal. is read "5 *times* 11 gallons"; 11 gal. $\times 5$ is read "11 gallons *multiplied by* 5"; 6×7 is read either "6 times 7" or "6 multiplied by 7."

The sign of multiplication is read *times* when it *precedes* the multiplicand, and *multiplied by* when it *follows* the multiplicand.

EXERCISES

52. Read and complete the following. Name the multiplicand, the multiplier, and the product. Describe each as concrete or abstract.

1. 3×7 days =

2. 2×16 ounces =

3. 3 feet $\times 12 =$

4. 15×2 pints =

5. 12 months $\times 2 =$

6. 4 pecks $\times 11 =$

7. 10×8 quarts =

8. 4 quarts $\times 13 =$

9. 12 inches $\times 9 =$

10. 20×24 hours =

11. How many days are there in 12 weeks?

MODEL SOLUTION

1 week = 7 days;
12 weeks = 12×7 days = 84 days.

Multiplicand and product denote days.

NOTE. — In the following exercises, the tables on pages 278–280 may be consulted, if necessary.

12. How many ounces are there in 3 pounds?

13. How many feet are there in 13 yards?

14. How many oranges are 8 dozen oranges?

15. How many square feet are there in 9 square yards?

16. A man had 8 rows of cherry trees with 8 trees in each row. How many cherry trees had he?

17. From each of 9 classes, 7 children are chosen to take part in a Christmas play. How many children take part in the play?

18. Edward saw 3 flocks of wild geese, with 15 geese in each flock. How many geese did he see?

19. A boy mailed 12 letters to England with 5-cent stamps. How much did all the stamps cost?

20. How many cents are there in 11 dimes?

21. How far does a train go in 3 hours at the rate of 30 miles an hour?

22. If it takes 17 minutes to go from one place to another by a subway car and twice as long by a surface car, how long will it take by the surface car?

23. Find the cost of a dozen fans at 5 cents each.

24. How many quarts are there in a bushel? in 2 bushels?

25. Laths are tied in bundles of 50 each. How many laths are there in 20 bundles?

26. If a canal boat is towed at the rate of 2 miles per hour, how far will it be towed in 24 hours?

Find the cost of:

27. 30 pairs of rabbits at 15¢ per pair.

28. 16 pairs of canvasback ducks at $2 per pair.

29. 34 boxes of Kiefer pears at $2 per box.

30. A 10-pound box of cherries at 14¢ per pound.

31. 12 pounds of candied cherries at 40¢ per pound.

32. A 25-pound box of apricots at 10¢ per pound.

33. Ten 4-lb. baskets of Catawba grapes at 12¢ per pound.

EXERCISES

53. Multiply:

1.	36	48	75	140	250	710
	2	2	2	2	3	3
2.	44	65	609	35	216	413
	3	3	3	4	4	4
3.	25	64	72	120	312	908
	5	5	5	5	5	5
4.	15	43	82	212	13	34
	6	6	6	6	7	7
5.	52	612	25	42	212	500
	7	7	8	8	8	8
6.	14	22	113	909	56	75
	9	9	9	9	10	10
7.	101	44	25	61	104	312
	10	11	11	11	11	11
8.	40	32	25	46	203	250
	12	12	12	12	12	12
9.	130	140	150	160	117	118
	2	3	4	5	2	2
10.	515	225	119	214	320	501
	4	4	2	5	12	11

BUSINESS PROBLEMS

54. **1.** Find the cost of sending a 22-word telegram from Kansas City to New York City at 50¢ for the first 10 words and 3¢ for each additional word.

2. The rate mentioned in exercise 1 is the day rate in either direction, and is briefly written 50–3. The night rate is 30–2. How much does it cost to send the same telegram at night?

3. How much less does the telegram cost at night?

4. How much cheaper is a 15-word telegram at a night rate of 40–3 than at a day rate of 60–4?

Find the cost of each telegram between New York City and the place named, at day rates; at night rates:

Place	Number of Words	Day Rate	Night Rate	Place	Number of Words	Day Rate	Night Rate
5. Hoboken	16	20–1	20–1	**17.** St. Paul	18	50–3	30–2
6. Boston *	18	25–2	25–1	**18.** Mobile *	25	50–3	30–2
7. Bangor *	22	25–2	25–1	**19.** Raleigh *	40	50–3	30–2
8. Hartford *	24	25–2	25–1	**20.** Savannah *	60	50–3	30–2
9. Newport *	26	25–2	25–1	**21.** Tampa *	19	60–4	40–3
10. Richmond	17	35–2	25–1	**22.** Topeka	30	60–4	40–3
11. Detroit	19	40–3	30–2	**23.** New Orleans	22	60–4	40–3
12. Toronto	20	40–3	30–2	**24.** Denver *	17	75–5	60–4
13. Chicago	23	40–3	30–2	**25.** Galveston	21	75–5	50–3
14. St. Louis	25	40–3	30–2	**26.** Seattle *	17	100–7	100–7
15. Cleveland	30	40–3	30–2	**27.** Portland *	19	100–7	100–7
16. Louisville	32	40–3	30–2	**28.** St. Johns	15	125–11	105–10

* Other cities in the same state have the same rate.

29. Find the cost of sending the following telegram, counting only the words of the actual message:

THE WESTERN UNION TELEGRAPH COMPANY

11.30 A.M. New York, July 10, 1906

To Mrs. William Steel,

525 West 12th St., Chicago, Ill.

Arrived	safe	steamship	Cedric.	Reach
Chicago	9olk	Street	Station	five
thirty	to-morrow.	Bring	friend	Hastings.
				John Steel.

☞ READ THE NOTICE AND AGREEMENT ON BACK ☜

30. Find the cost of sending this telegram at 9 P.M.

31. How much will it cost to talk over a long-distance telephone for 7 minutes at 25¢ for the first 3 minutes and 5¢ for each additional minute (25–5)?

32. Alfred talked with his father in a distant city from 7:55 P.M. to 8:03 P.M. Find the telephone charges at 50–15.

33. How much would the conversation have cost by day at 100–25?

Find the cost of telephoning for:

34. 10 min. @ 20–5.

35. 11 min. @ 40–10.

36. 12 min. @ 50–10.

37. 6 min. @ 60–15.

38. 8 min. @ 100–25.

39. 5 min. @ 150–40.

55. 1. Multiply 2 by 10; by 100; by 1000.

2. Multiply 25 by 10; by 100; by 1000.

3. Multiply 156 by 100; by 1000; by 10,000.

4. How may an integer be multiplied by 10? by 100? by 1000? by 10,000? by 100,000?

An integer may be multiplied by 10, 100, 1000, *etc., by annexing to it as many ciphers as there are ciphers in the multiplier.*

WRITTEN EXERCISES

56. Multiply by 10, by 100, by 1000, and by 100,000:

1. 32	5. 506	9. 247	13. 210
2. 56	6. 708	10. 396	14. 400
3. 85	7. 124	11. 101	15. 7386
4. 79	8. 365	12. 308	16. 5490

Multiply:

17.	18.	19.	20.	21.
234	234	725	408	760
20	200	120	800	900
4680	***00	***0	****00	***00

Multiply:

22. 38 by 20	28. $125 by 400	34. 165 by 3000
23. 58 by 70	29. $632 by 500	35. 287 by 9000
24. 49 by 30	30. $496 by 600	36. 429 by 8000
25. 75 by 90	31. $328 by 800	37. 360 by 4000
26. $46 by 110	32. $560 by 110	38. 798 by 7000
27. $93 by 120	33. $784 by 120	39. 590 by 1200

40. Find the area of a tennis court 78 ft. long and 30 ft. wide.

41. A barrel of flour weighs 196 pounds. How many pounds of flour are required to fill 40 barrels?

42. If the heart beats 72 times in a minute, how many times does it beat in an hour?

43. A barge ferries 16 cars over the Hudson River in one trip. How many does it ferry over in 20 trips?

44. A certain large city consumes 400 car loads of fruit and vegetables per day. How many tons are consumed in a day, if a car load averages 15 tons?

57. 1. What part of 10 yards is 5 yards?
What part of 10 times 3 feet is 5 times 3 feet?
Compare 5 times 4 with 10 times 4; 5×8 with 10×8.

2. How is 10×48 found? How may 5×48 be found from 10×48? $5 \times 48 = ?$

$$2\overline{)480}\quad 240$$

3. Tell an easy way to multiply any integer by 5.

4. How is 100×66 found? How may 50×66 be found from 100×66? $50 \times 66 = ?$
How may any integer be multiplied by 50?

$$2\overline{)6600}\quad 3300$$

5. What part of 1000 is 500? How is any integer multiplied by 1000? How, then, may any integer be multiplied by 500? $500 \times 24 = ?$ $500 \times 18 = ?$

6. Since any integer may be multiplied by 100 by annexing two ciphers, and since 25 is $\frac{1}{4}$ of 100, how may any integer be multiplied by 25? $25 \times 44 = ?$ $25 \times 32 = ?$

7. Since any integer may be multiplied by 1000 by annexing three ciphers, and since 250 is $\frac{1}{4}$ of 1000, how may any integer be multiplied by 250? $250 \times 44 = ?$ $250 \times 16 = ?$

$$4\overline{)44{,}000}\quad 11{,}000$$

58. *Any integer may be multiplied*
By 50, *by annexing two ciphers and dividing by* 2.
By 500, *by annexing three ciphers and dividing by* 2.
By 25, *by annexing two ciphers and dividing by* 4.
By 250, *by annexing three ciphers and dividing by* 4.

WRITTEN EXERCISES

59. Multiply by 5, by 50, by 500, by 25, by 250:

1. 64	**7.** 90	**13.** 202	**19.** 990
2. 86	**8.** 220	**14.** 814	**20.** 870
3. 98	**9.** 160	**15.** 735	**21.** 1125
4. 69	**10.** 128	**16.** 916	**22.** 3487
5. 71	**11.** 177	**17.** 497	**23.** 4966
6. 87	**12.** 346	**18.** 318	**24.** 9999

25. How much will it cost to rent a house for 5 years at $1800 a year?

26. A barrel of salt weighs 280 pounds. Find the weight of 25 barrels of salt.

27. A field yielded 50 bales of cotton averaging 496 pounds each. How many pounds of cotton did it yield?

28. A bushel of shelled corn weighs 56 pounds. Find the weight of 500 bushels.

WRITTEN EXERCISES

60.

1.	**2.**	**3.**	**4.**
95	4387	3675	350
350	1204	2008	95
4750	17548	29400	1750
285	52644	7350	3150
33250	5281948	7379400	33250

95 multiplied by 5 tens = 475 *tens*.

4387 multiplied by 12 hundreds = 52644 *hundreds*.

3675 multiplied by 2 thousands = 7350 *thousands*.

Exercise 4 shows the method of testing the answer in exercise 1. Test the other answers.

Multiply and test:

5. 345 × 240	6. 427 × 990	7. 538 × 705	8. 932 × 1004
9. 89 × 4400	10. 488 × 1201	11. 843 × 480	12. 999 × 1212

13. 409×45	21. 607×304	29. 7854×1210
14. 790×68	22. 418×730	30. 4936×8020
15. 480×99	23. 922×908	31. 3705×5550
16. 650×85	24. 756×709	32. 4010×8750
17. 396×430	25. 1244×202	33. $75{,}073 \times 4800$
18. 589×280	26. 1201×837	34. $49{,}000 \times 3609$
19. 278×760	27. 1108×746	35. $55{,}410 \times 5040$
20. 169×970	28. 1010×890	36. $37{,}962 \times 7588$

Multiply:

37. \$85.25 by 48	41. \$110.75 by 92
38. \$56.62 by 36	42. \$434.19 by 206
39. \$83.69 by 77	43. \$391.72 by 508
40. \$42.96 by 29	44. \$88.93 by 2505

WRITTEN EXERCISES

61. 1. How far will a train go in 24 hours at the rate of 40 miles an hour?

24
40
960, number of miles.

SOLUTION. — Since the train goes 40 miles an hour, in 24 hours it will go 24 times 40 miles.

Since $24 \times 40 = 40 \times 24$, the number of miles the train will go may be found either by multiplying 40 by 24, or by multiplying 24 by 40.

Since it is easier to multiply by 40 than by 24, the latter method is used.

2. Find the number of pounds of cheese in 175 cheeses that weigh 40 pounds each.

3. Find the weight of 126 bales of cotton, if the average weight of a bale is 500 pounds.

4. At the bank a boy saw 18 bags, each of which contained $5000 in gold coin. Find the value of the gold in all the bags.

5. A grocer bought 185 cases of eggs. Each case contained 30 dozen. How many dozen eggs did he buy?

6. A coffee plantation of 62 acres contained 500 trees to the acre. Find the whole number of coffee trees.

7. Find the value of 219 Angora goats at $5.75 a head.

8. The clip from a flock of Angora goats was 847 pounds of mohair. Find its value at $.35 per pound.

9. The material for a dress was 14 yards of mohair and cost $1.25 per yard. Making and trimmings cost $18.75. Find the cost of the dress.

10. An upholsterer bought 75 yards of mohair plush at $1.50 per yard. Find the cost.

11. How much must a harness maker pay for 36 mohair carriage robes at $22 each?

12. How much will it cost to drill a well 1125 feet deep at $1.05 per foot?

13. A cargo of lemons from Sicily consisted of 25,000 boxes each weighing 92 pounds. Find the weight of the cargo.

14. A jobber bought 225 boxes of lemons at $2.85 a box. Find the cost of his purchase.

How much will it cost to send a message by cable from

15. New York to London, 15 words at $.25 per word?

16. New York to Tokyo, 18 words at $1.53 per word?

17. New York to Manila, 24 words at $1.12 per word?

18. New York to Constantinople, 88 words at $.37 per word?

DIVISION

62. 1. How many times does 150 contain 15? How many times does 152 contain 15, and how many units remain?

2. Separate 150 units into 10 equal parts. How many units are there in each part?

3. If 155 is separated into 10 equal parts, how many units and parts of a unit will there be in each part?

63. The process of finding how many times one number contains another, or the process of separating a number into equal parts, is called **division.**

64. The number divided is called the **dividend.**

65. The number by which we divide is called the **divisor.**

66. The number obtained by dividing is called the **quotient.**

67. The part of the dividend remaining when the division is not exact is called the **remainder.**

DRILL EXERCISES

68. Tell quotients instantly:

32 ÷ 8	40 ÷ 8	56 ÷ 7	72 ÷ 12	120 ÷ 12
42 ÷ 6	45 ÷ 5	54 ÷ 6	60 ÷ 12	110 ÷ 11
30 ÷ 6	42 ÷ 7	56 ÷ 8	81 ÷ 9	108 ÷ 12
35 ÷ 7	40 ÷ 5	60 ÷ 5	99 ÷ 9	132 ÷ 12
24 ÷ 8	45 ÷ 9	64 ÷ 8	84 ÷ 12	100 ÷ 10
28 ÷ 7	48 ÷ 6	63 ÷ 7	84 ÷ 7	121 ÷ 11
36 ÷ 4	49 ÷ 7	72 ÷ 8	96 ÷ 8	120 ÷ 10
35 ÷ 5	54 ÷ 9	72 ÷ 6	96 ÷ 12	132 ÷ 11
36 ÷ 9	36 ÷ 6	72 ÷ 9	99 ÷ 11	144 ÷ 12

69. **1.** \$36 may be divided by \$4, but not by 4 years, 4 gallons, 4 pounds, etc.

The dividend and divisor, if concrete, must be like numbers.

2. Divide \$36 by \$4; 60 gal. by 12 gal.; 42 days by 7 days; 56 in. by 8 in.

When the divisor is like the dividend, the quotient is abstract.

3. Divide \$96 by 12; 72 hr. by 8; 81¢ by 9.

When the dividend is concrete and the divisor is abstract, the quotient is like the dividend.

4. $$21 \text{ days} \div 7 \text{ days} = 3$$

means that 21 days contains 7 days 3 times.

$$21 \text{ days} \div 3 = 7 \text{ days}$$

means that $\frac{1}{3}$ of 21 days is 3 days.

The sign ÷ may denote either measuring or dividing into parts, but in either case it is read "**divided by.**"

EXERCISES

70. Divide:

1. 64 qt. ÷ 8

2. 48 lb. ÷ 3

3. 24 hr. ÷ 12 hr.

4. 32 oz. ÷ 16 oz.

5. 700¢ ÷ 100¢.

6. 34 pt. ÷ 17

7. 96 ft. ÷ 12

8. 45 days ÷ 15

9. 72 in. ÷ 12 in.

10. 42 bu. ÷ 14 bu.

11. How many weeks are there in 365 days?

SOLUTION

7 days)365 days

52 times, + 1 day

52 wk. 1 day

Since 7 days = 1 wk., 365 days are equal to as many weeks as 7 days are contained times in 365 days.

Then, 365 days = 52 wk. 1 day = 52$\frac{1}{7}$ wk.

NOTE. — Consult the tables on pages 278–280 if necessary.

12. How many pounds are there in 160 ounces?

13. How many yards are there in 34 feet?

14. A box contains 126 oranges. How many dozen oranges does it contain?

15. If 45 cents is divided among 3 boys, how many cents will each boy receive?

16. If an 18-acre field yields 36 tons of clover, how many tons per acre does it yield?

17. How many times does the hour hand of a clock go around the clock face in 72 hours?

18. How many bushels are there in 50 pecks?

19. If a motor car runs 12 miles an hour, how long will it take to run 100 miles?

20. Four boys weigh 283 pounds. Find the average weight.

21. A famous cherry tree in California yielded, in five years, 2000 lb., 1800 lb., 3000 lb., 2200 lb., and 3000 lb. of cherries. Find the average annual yield.

EXERCISES

71. The expressions $\$22 \div \$5$, $\$5)\underline{\$22}$, and $\frac{\$22}{\$5}$ all indicate that \$22 is to be divided by \$5.

The answer should be given in the form:

"4 and \$2 remainder, or $4\frac{2}{5}$."

Divide:

1. $132 \div 2$	**4.** $\$180 \div 6$	**7.** $\$5)\underline{\$510}$	**10.** $8)\underline{\$404}$
2. $632 \div 7$	**5.** $\$810 \div 9$	**8.** $\$3)\underline{\$132}$	**11.** $7)\underline{\$364}$
3. $722 \div 9$	**6.** $\$132 \div 12$	**9.** $\$5)\underline{\$351}$	**12.** $11)\underline{999}$
13. $\frac{90\text{¢}}{6\text{¢}}$	**14.** $\frac{341 \text{ wk.}}{7}$	**15.** $\frac{\$282}{4}$	**16.** $\frac{\$639}{9}$

WRITTEN EXERCISES

72. Divide:

17. $\$729 \div \$8$	18. $\$9656 \div 8$	19. 896 yd. $\div 8$
20. $12\overline{)52800}$	24. $11\overline{)\$8877}$	28. $8\overline{)\$75.94}$
21. $11\overline{)49390}$	25. $10\overline{)\$10.10}$	29. $7\overline{)\$87.43}$
22. $12\overline{)\$76.80}$	26. $12\overline{)\$98.40}$	30. $9\overline{)\$89.91}$
23. $\$6\overline{)\$5914}$	27. $\$11\overline{)\$4004}$	31. $6\overline{)\$12108}$
32. $\frac{168 \text{ hr.}}{7 \text{ hr.}}$	35. $\frac{110 \text{ sq. in.}}{5 \text{ sq. in.}}$	38. $\frac{1320 \text{ cu. in.}}{12 \text{ cu. in.}}$
33. $\frac{1760 \text{ yd.}}{11}$	36. $\frac{5280 \text{ ft.}}{8}$	39. $\frac{5760 \text{ oz.}}{12 \text{ oz.}}$
34. $\frac{7249 \text{ sq. ft.}}{9 \text{ sq. ft.}}$	37. $\frac{3000 \text{ mi.}}{8}$	40. $\frac{43360 \text{ in.}}{12 \text{ in.}}$

73. 1. Divide 20, 60, 210, 350, 600, and 7000 by 10.

How may an integer ending in one or more ciphers be divided by 10?

2. Divide 300, 800, 1100, 2500, and 16,000 by 100.

How may an integer ending in two or more ciphers be divided by 100?

3. Divide 11,000, 225,000, and 3,000,000 by 1000.

Make a rule about dividing certain integers by 1000.

4. Divide 5762 by 10; by 100; by 1000.

What is the remainder in each case?

An integer may be divided by 10, 100, 1000, *etc., by cutting off as many figures from the right as there are ciphers on the right of the divisor. The figures cut off form the remainder if there is any.*

EXERCISES

74. Divide by 10, by 100, by 1000, and by 100,000:

1. 5,200,000	**5.** 9,246,340	**9.** 11,482,000
2. 7,568,000	**6.** 2,756,982	**10.** 58,700,000
3. 4,300,000	**7.** 7,536,289	**11.** 90,000,000
4. 8,000,000	**8.** 4,750,304	**12.** 84,300,750

WRITTEN EXERCISES

75. Divide:

1. $2\|0\overline{)86\|0}$ 43	**2.** $4\|0\overline{)36\|7}$ $9\frac{7}{40}$	**3.** $3\|00\overline{)15\|23}$ $5\frac{23}{300}$
4. $30\overline{)9300}$	**5.** $60\overline{)4260}$	**6.** $800\overline{)24800}$
7. $40\overline{)1660}$	**8.** $50\overline{)3550}$	**9.** $700\overline{)56700}$
10. $80\overline{)8831}$	**11.** $600\overline{)1271}$	**12.** $4000\overline{)24511}$
13. $70\overline{)8493}$	**14.** $200\overline{)1837}$	**15.** $5000\overline{)45313}$
16. $90\overline{)1087}$	**17.** $800\overline{)9633}$	**18.** $9000\overline{)99919}$

19. Divide 2457 by 40.

$4|0\overline{)245|7}$
$61\frac{17}{40}$

4 tens are contained in 245 tens, 61 times, with 1 ten remainder.

Therefore 40 is contained in 2450, 61 times, with 1 ten remainder; and in 2457, 61 times with 1 ten and 7 units, or 17, remainder.

Divide by 20, 30, 40, 50, 60, 70, 80, and 90:

20. 3433	**22.** 6257	**24.** 5827	**26.** 60,077
21. 1271	**23.** 4739	**25.** 3271	**27.** 97,001

Divide by 200, 300, 400, 500, 600, 700, 800, and 900:

28. 7523	**30.** 42,641	**32.** 73,099	**34.** 356,101
29. 6931	**31.** 38,407	**33.** 65,411	**35.** 701,407

36. A 40-acre vineyard in California produced grapes worth $4160. Find the value of the crop per acre.

37. How long will it take a fast freight train running 20 miles an hour to go from Charleston to New York, 804 miles?

38. Mr. Hoy bought a car load of grain. It weighed 34,000 pounds. How many tons did it weigh?

39. A car that weighed 22,800 pounds when empty was loaded with wheat. It then weighed 56,460 pounds. How many bushels of wheat did it contain, if 1 bushel weighs 60 pounds?

40. How much are the freight charges on 35,000 pounds of machinery sent from St. Louis to Wichita at 52¢ per 100 pounds?

$.52
350ØØ
2600
156
$182.00

Dividing 35,000 pounds by 100 pounds by cutting off the last two ciphers of 35,000, it is found that 35,000 pounds = 350 hundredweight. Since the freight on 1 hundredweight is $.52, the freight on 350 hundredweight is 350 times $.52, or $182.

Freight rates are usually quoted per 100 pounds.

Find the freight charges on:

41. 23,200 pounds brick @ 22¢.

42. 45,000 pounds sugar @ 28¢.

43. 36,300 pounds coffee @ 35¢.

44. 28,900 pounds rice @ 38¢.

45. 56,400 pounds molasses @ 75¢.

46. 38,400 pounds cotton piece goods @ 92¢.

47. Find the freight charges on 88,600 lb. of sugar shipped from New Orleans to Minneapolis @ 33¢; from New Orleans to Salt Lake City @ $1.50.

48. Find the freight on 776 400 lb. of corn shipped from Omaha to New York City @ 20¢.

49. Find the freight on 27,000 lb. of raisins shipped from Stockton, Cal., to New York at 95¢.

50. Find the freight on 1400 crates of strawberries shipped from Wilmington, N.C., to Pittsburg at 65¢ per crate.

Find the cost of:

51. 22,000 bricks at $6.50 per 1000.

52. 18,000 strawberry plants at $2.50 per 1000.

53. 44,000 cu. ft. of gas at $.95 per 1000 cu. ft.

54. 84,600 lb. of bar iron at $2.10 per 100 lb.

WRITTEN EXERCISES

76.

1.	2.	3.
245	$6.44	5063
764)187180	275)$1771.00	427)2161901
1528	1650	2135
3438	1210	2690
3056	1100	2562
3820	1100	1281
3820	1100	1281

Divide:

4.	8544 by 24	**13.**	17,088 by 356
5.	1722 by 42	**14.**	$102.60 by 285
6.	64,260 by 85	**15.**	$577.98 by 234
7.	71,916 by 78	**16.**	$360.36 by 156
8.	62,923 by 89	**17.**	158,976 by 414
9.	$63,595 by 79	**18.**	246,906 by 522
10.	$608.16 by 56	**19.**	553,491 by 691
11.	$899.91 by 99	**20.**	348,425 by 385
12.	$90,600 by 75	**21.**	645,582 by 798

Divide by 658; 782; 496; 879; 1025; 4560; 7854:

22. 594,832	**26.** 358,632	**30.** 6,597,774
23. 316,710	**27.** 930,040	**31.** 3,385,200
24. 314,675	**28.** 792,232	**32.** 4,775,232
25. 450,864	**29.** 848,470	**33.** 8,458,758

34. How many cars of 48 tons' capacity are required to receive the cargo of a Lake Erie vessel carrying 8400 tons of iron ore?

35. If a bale of hops weighs 180 lb., how many bales and how many pounds over will make a ton?

36. It cost a man $1275.00 to drill a gas well 1500 ft. deep. How much did the work cost per foot?

37. A rolling mill consumes about 3,500,000 cu. ft. of natural gas per day. How many households would this amount supply, if each uses 875 cu. ft. per day?

38. A spool of barbed wire contains 1500 ft. How much does this length of wire weigh, if 12 ft. weigh 1 lb.?

39. A 75-acre field yielded 3 crops of alfalfa in a year, the first 187 tons, the second 158 tons, and the third 105 tons. Find the annual yield per acre.

40. A steamship has 2450 tons of coal in her bunkers. If she burns 175 tons per day, how long will the coal last?

41. It required 160 dollars' worth of gypsum to fertilize a hop field 80 rd. by 40 rd. Find the cost per acre.

42. A New England state road cost $6388.80 per mile. How much did it cost per lineal foot?

43. A strawberry farm in New Jersey yielded 140,000 qt. of strawberries. How many bushel crates did it yield?

44. In one season 69,750 crates of strawberries were shipped from one county in Delaware. Find the number of car loads shipped, considering 225 crates as a car load.

FACTORING

77. 1. Tell two numbers whose product is 12; 15; 18; 21; 25; 28; 30; 60; 100.

2. Of what three numbers is 12 the product? 18? 28?

3. Of what four numbers is 16 the product? 60?

4. Changing 12 to 2×6, or to $2 \times 2 \times 3$, is called *factoring* 12. The factors of 12 are 2 and 6 or 2, 2, and 3.

78. The integers that multiplied together produce a given number are called its **factors.**

79. The process of separating a number into its factors is called **factoring.**

80. 1. What numbers are exactly contained in 12, or are *exact divisors* of 12?

2. If 2 is taken as one of two factors of 18, what is the other factor? How is this second factor found?

3. One factor of 39 is 3. What is the other factor?

81. *An exact divisor of a number is a factor of it.*

In separating a number into two factors, any exact divisor may be taken for one factor and the quotient for the other.

82. 1. Name all the exact divisors or factors of 14, including 14 and 1; of 10; of 12; of 18.

2. Name all the factors of 5; of 7; of 19.

83. A number that has no factors except itself and 1 is called a **prime number.**

84. A number that has other factors than itself and 1 is called a **composite number.**

85. 1. Tell which of the following numbers exactly contain 2, or are *divisible* by 2:

2, 4, 6, 10, 11, 13, 15, 18, 21, 30, 36, 41, 50

2. Which of them are not divisible by 2?

86. A number that is divisible by 2 is called an **even number.** A number that is not divisible by 2 is called an **odd number.**

0 is regarded as an even number, 1 as an odd number.

EXERCISES

87. 1. Tell all the exact divisors of numbers from 1 to 50.

2. Separate into two factors in all possible ways:

10, 12, 16, 21, 32, 36, 24, 28, 40, 42

3. Tell whether each of the following is prime or composite, and why; also whether it is even or odd, and why:

1, 7, 6, 9, 10, 11, 12, 13, 14, 19, 21, 23, 27, 29

4. Find the prime factors of:

4	6	14	15	24	36	42
9	10	12	22	28	32	48

Tests of Divisibility

88. 1. Make a list of numbers from 2 to 50 that contain 2 as a factor. Notice the units' figure of each.

2. Make a list of numbers from 5 to 100 that contain 5 as a factor. What must the units' figure of such a number be?

3. Make a list of numbers from 3 to 60 that contain 3 as a factor. Find whether 3 is exactly contained in the sum of the figures, or *digits*, of each number.

4. Make a list of numbers from 9 to 90 that contain 9 as a factor. Find whether 9 is contained exactly in the sum of the digits of each number.

89. *A number is divisible by*

2, *if the units' figure is* 2, 4, 6, 8, *or* 0.

5, *if the units' figure is* 5 *or* 0.

3, *if the sum of the digits is divisible by* 3.

9, *if the sum of the digits is divisible by* 9.

EXERCISES

90. **1.** Select the numbers that are divisible by 2:

36	926	426	1600	1235	7644
84	345	866	1440	6786	5373
25	723	792	1385	4041	6030

2. Which of the foregoing numbers are divisible by 5?

3. Which are divisible by 3?

4. Which are divisible by 9?

WRITTEN EXERCISES

91. **1.** Find the prime factors of 378.

$$\begin{array}{r|l} 2 & 378 \\ 9 & 189 \\ 3 & 21 \\ & 7 \end{array}$$

$378 = 2 \times 3 \times 3 \times 3 \times 7$

Since the last figure of 378 is divisible by 2, what number is a factor of 378? Dividing by 2, what is the other factor?

Since the sum of the digits of 189 is divisible by 9, what number is a factor of 189? Dividing by 9, what is the other factor?

Since $21 = 3 \times 7$ and the factors of 378 previously obtained were 2 and 9, or 2, 3, and 3, $378 = 2 \times 3 \times 3 \times 3 \times 7$.

2. Find the prime factors of 1485; of 300; of 840.

$$\begin{array}{r|l} 5 & 1485 \\ 9 & 297 \\ 3 & 33 \\ & 11 \end{array}$$

$1485 = 5 \times 3 \times 3 \times 3 \times 11$

$$\begin{array}{r|l} 10 & 300 \\ 10 & 30 \\ & 3 \end{array}$$

$300 = 2 \times 5 \times 2 \times 5 \times 3$

$$\begin{array}{r|l} 10 & 840 \\ 4 & 84 \\ 3 & 21 \\ & 7 \end{array}$$

$840 = 5 \times 2 \times 2 \times 2 \times 3 \times 7$

Find the prime factors of:

3. 160	13. 126	23. 111	33. 6250
4. 220	14. 168	24. 123	34. 2220
5. 480	15. 198	25. 234	35. 2016
6. 210	16. 672	26. 345	36. 1375
7. 192	17. 576	27. 456	37. 1296
8. 270	18. 324	28. 567	38. 1024
9. 216	19. 801	29. 648	39. 1728
10. 225	20. 495	30. 780	40. 13,068
11. 180	21. 363	31. 891	41. 80,000
12. 375	22. 441	32. 912	42. 10,692

Cancellation

92. 1. How many times is 2 times 5 contained in 4 times 5? 2×6 in 4×6? 2 times any number in 4 times that number?

2. How many times is 3×10 contained in 6×10? 3×25 in 6×25? 3 times any number in 6 times that number?

3. How many times is 6×7 contained in 12×7? 5×8 in 15×8? 9×150 in 36×150?

4. Divide 48×11 by 8×11. $8 \times \cancel{11})\overline{48 \times \cancel{11}}$ 6

What factor may be omitted, or *canceled*, from both dividend and divisor?

5. In dividing 77×16 by 7×16 what factor may be canceled from dividend and divisor? in dividing 12×12 by 12×4?

6. Divide 72×13 by 9×13; 31×24 by 31×6; 42×19 by 7; 64×11 by 8; 9×25 by 3×5.

93. *Canceling equal factors from dividend and divisor, or dividing dividend and divisor by the same number, does not change the quotient.*

WRITTEN EXERCISES

94. 1. Divide $21 \times 5 \times 16 \times 44$ by $7 \times 25 \times 32 \times 11$.

$$\frac{\overset{3}{\cancel{21}} \times \overset{1}{\cancel{5}} \times \overset{2}{\cancel{16}} \times \overset{4}{\cancel{44}}}{\underset{5}{\cancel{7}} \times \underset{2}{\cancel{25}} \times \cancel{32} \times \cancel{11}} = \frac{6}{5} = 1\tfrac{1}{5}$$

The dividend is written above the divisor with a line between them.

Dividing the dividend and divisor by 7 does not change the value of the quotient, hence the factor 7 may be canceled from 21 in the dividend, and from 7 in the divisor, leaving in their places 3 in the dividend and 1 in the divisor.

Next, the dividend and divisor may be divided by 5; then by 16; then by 11; and finally by 2, a factor of 4 and 2.

The remaining factors of the dividend are 3, 1, 1, and 2; of the divisor, 1, 5, 1, and 1. The quotient is $\frac{6}{5}$ or $1\frac{1}{5}$.

NOTES. — 1. When the process of cancellation is thoroughly understood, the 1's need not be written in dividend or divisor.

2. If all the factors of the dividend are canceled, the resulting dividend is 1. The same is true of the divisor. If all the factors of both dividend and divisor are canceled, the quotient is 1.

Divide:

2. $8 \times 5 \times 30 \times 16 \times 15$ by $4 \times 10 \times 6 \times 12 \times 3$

3. $15 \times 48 \times 70 \times 11 \times 40$ by $30 \times 16 \times 7 \times 22 \times 50$

4. $17 \times 9 \times 12 \times 11 \times 28$ by $34 \times 6 \times 72 \times 7 \times 22$

5. $42 \times 35 \times 56 \times 4 \times 12$ by $28 \times 49 \times 14 \times 10 \times 96$

6. $3 \times 17 \times 48 \times 4 \times 65$ by $5 \times 34 \times 13 \times 24 \times 12$

7. $63 \times 36 \times 48 \times 27 \times 96 \times 69$ by $81 \times 81 \times 144 \times 46$

8. $48 \times 48 \times 20 \times 65 \times 36 \times 54$ by $2880 \times 1080 \times 936$

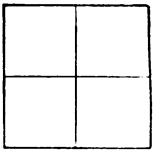
9. $792 \times 240 \times 35 \times 756$ by $882 \times 180 \times 800 \times 33$

10. $512 \times 27 \times 720 \times 847$ by $64 \times 144 \times 99 \times 55$

11. $168 \times 144 \times 216 \times 432 \times 484$ by $224 \times 198 \times 576 \times 162$

12. $729 \times 420 \times 147 \times 78 \times 88$ by $162 \times 63 \times 231 \times 96 \times 260$

FRACTIONS



95. 1. Into how many parts is this square divided? How do the parts compare in size?

2. What name is given to one of the four equal parts of anything? to three of the four equal parts?

3. If any one thing is divided into five equal parts, what name is given to one part? to two parts? to three parts? to four parts?

96. Any one thing is called a **unit.**

97. One or more of the equal parts of a unit is called a **fraction.**

98. 1. Two numbers are used to write a fraction — one above and the other below a line; thus, three fourths is written $\frac{3}{4}$.

Write four fifths, five sixths, seven eighths.

2. In the fraction $\frac{3}{4}$, which number shows into how many equal parts the thing, or unit, is divided? which shows how many parts form the fraction?

3. In the fraction $\frac{4}{5}$, what does the 5 show? the 4? What does the 8 show in the fraction $\frac{7}{8}$? the 7?

99. The number that shows into how many equal parts the unit is divided is called the **denominator.**

It is written below the line.

100. The number that shows how many parts form a fraction is called the **numerator.**

It is written above the line.

101. The numerator and denominator of a fraction are called its **terms**.

What are the terms of the fraction $\frac{2}{3}$? Which is the numerator? the denominator?

EXERCISES

102. **1.** Read $\frac{9}{206}$, $12\frac{3}{20}$, $\$2.66\frac{2}{3}$.

$\frac{9}{206}$ is read, "nine two-hundred-sixths."
$12\frac{3}{20}$ is read, "twelve and three twentieths."
$\$2.66\frac{2}{3}$ is read, "two dollars, sixty-six and two-thirds cents."

Read:

2. $\frac{16}{25}$ **5.** $\frac{37}{160}$ **8.** $\frac{89}{200}$ **11.** $34\frac{7}{8}$ in. **14.** $\$5.12\frac{1}{2}$

3. $\frac{19}{42}$ **6.** $\frac{184}{231}$ **9.** $\frac{111}{1728}$ **12.** $42\frac{5}{16}$ lb. **15.** $\$18.33\frac{1}{3}$

4. $\frac{22}{61}$ **7.** $\frac{71}{320}$ **10.** $26\frac{3}{19}$ **13.** $78\frac{21}{25}$ T. **16.** $\$85.46\frac{3}{4}$

WRITTEN EXERCISES

103. Write in figures:

1. Seventy-five eightieths.
2. Nineteen fifty-seconds.
3. Twenty-five hundredths.
4. Eleven four-hundredths.
5. Fourteen thousandths.
6. Sixty-eight ninetieths.
7. Eighty-two thirty-sixths.
8. Five and seven eighths.
9. Five hundred two-thousandths.
10. Seventy-seven one-hundred-fortieths.
11. Nine hundred four six-thousand-two-hundred-tenths.
12. Two hundred one and twenty one-hundred-fourths.
13. Sixty-four dollars forty-two and one-fourth cents.

Write in words:

14. $\frac{25}{48}$ **17.** $\frac{14}{101}$ **20.** $\frac{175}{26\,81}$ **23.** $69\frac{19}{61}$ **26.** $\$46.62\frac{1}{2}$

15. $\frac{68}{54}$ **18.** $\frac{37}{144}$ **21.** $\frac{288}{1728}$ **24.** $86\frac{21}{37}$ **27.** $\$125.33\frac{1}{3}$

16. $\frac{73}{90}$ **19.** $\frac{496}{217}$ **22.** $\frac{462}{8249}$ **25.** $50\frac{36}{75}$ **28.** $\$376.24\frac{5}{8}$

Reduction of Fractions

104. 1. To how many inches are 3 feet equal?

Change 2 pounds to ounces; 8 quarts to gallons.

2. How many fourths are there in 1? in $\frac{1}{2}$?

Change $\frac{6}{8}$ to fourths; $\frac{1}{8}$ to sixths.

3. How many thirds are there in 4? in $4\frac{2}{3}$?

Change $1\frac{0}{2}$ to ones; $1\frac{3}{5}$ to ones and fifths.

105. The process of changing the form of a number without changing its value is called **reduction.**

106. Reduction of fractions to higher or lower terms.

1. How many fourths are there in $\frac{1}{2}$? how many eighths?

How may the terms of $\frac{2}{4}$ be obtained from those of $\frac{1}{2}$? the terms of $\frac{4}{8}$ from those of $\frac{1}{2}$? from those of $\frac{2}{4}$?

Which has the larger, or *higher*, terms, $\frac{1}{2}$ or $\frac{2}{4}$? $\frac{2}{4}$ or $\frac{4}{8}$?

2. Change $\frac{1}{2}$ to sixths; to twelfths. Change $\frac{2}{3}$ to sixths; to ninths. Change $\frac{3}{4}$ to eighths.

3. How many eighths are there in $\frac{12}{16}$? how many fourths?

How may the terms of $\frac{6}{8}$ be obtained from those of $\frac{12}{16}$? the terms of $\frac{3}{4}$ from those of $\frac{6}{8}$? from those of $\frac{12}{16}$?

Which has the smaller, or *lower*, terms, $\frac{12}{16}$ or $\frac{6}{8}$? $\frac{6}{8}$ or $\frac{3}{4}$?

What fraction expresses $\frac{12}{16}$ in its *lowest* terms?

4. Reduce $\frac{4}{8}$ to halves; $\frac{2}{6}$ to thirds; $\frac{6}{10}$ to fifths; $\frac{10}{12}$ to sixths; $\frac{14}{16}$ to eighths.

107. *Multiplying or dividing both terms of a fraction by the same number does not change its value.*

108. A number that will exactly divide two or more numbers is called a **common divisor** of those numbers.

109. A fraction is expressed in its **lowest terms** when its terms have no *common divisor* except 1.

EXERCISES

110. 1. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$.

2. Change $\frac{1}{2}$ to tenths; $\frac{2}{3}$ to eighteenths; $\frac{4}{5}$ to twentieths.

3. Change to twenty-fourths: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{11}{12}$.

4. Reduce $\frac{4}{8}$ to halves; $\frac{4}{12}$ to thirds; $\frac{9}{12}$ to fourths.

5. Reduce to lower terms: $\frac{3}{9}$; $\frac{2}{10}$; $\frac{8}{12}$; $\frac{7}{14}$; $\frac{10}{15}$; $\frac{8}{16}$.

6. Reduce to lowest terms: $\frac{4}{8}$; $\frac{6}{12}$; $\frac{8}{16}$; $\frac{12}{16}$; $\frac{10}{20}$; $\frac{16}{24}$.

WRITTEN EXERCISES

111. 1. Reduce $\frac{5}{8}$ to thirty-seconds.

$\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$

We have learned that it will not change the value of $\frac{5}{8}$ to multiply both terms by any number. $32 \div 8 = 4$; then we multiply both terms by 4.

2. Reduce to forty-eighths: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{7}{8}$; $\frac{5}{12}$; $\frac{9}{16}$.

3. Reduce to sixtieths: $\frac{1}{3}$; $\frac{1}{4}$; $\frac{2}{5}$; $\frac{5}{6}$; $\frac{3}{10}$; $\frac{7}{12}$; $\frac{8}{15}$; $\frac{11}{20}$.

4. Reduce $\frac{45}{60}$ to its lowest terms.

$\frac{5)45}{5)60} = \frac{9}{12}$

$\frac{3)9}{3)12} = \frac{3}{4}$

We have learned that it will not change the value of $\frac{45}{60}$ to divide both terms by any number. By § 89 we see that 5 and also 3 will exactly divide both terms of $\frac{45}{60}$. Dividing both terms by 5, $\frac{45}{60} = \frac{9}{12}$; dividing the terms of $\frac{9}{12}$ by 3, $\frac{9}{12} = \frac{3}{4}$.

The terms of $\frac{3}{4}$ have no common divisor except 1; then $\frac{45}{60}$ reduced to lowest terms is equal to $\frac{3}{4}$.

Reduce to lowest terms:

5. $\frac{18}{36}$ 8. $\frac{24}{40}$ 11. $\frac{25}{125}$ 14. $\frac{126}{315}$ 17. $\frac{375}{1000}$

6. $\frac{15}{27}$ 9. $\frac{32}{60}$ 12. $\frac{84}{280}$ 15. $\frac{144}{576}$ 18. $\frac{288}{1728}$

7. $\frac{20}{60}$ 10. $\frac{18}{72}$ 13. $\frac{196}{224}$ 16. $\frac{630}{735}$ 19. $\frac{875}{2000}$

20. Reduce to fortieths: $\frac{1}{2}$; $\frac{3}{4}$; $\frac{2}{5}$; $\frac{3}{8}$; $\frac{7}{10}$; $\frac{34}{80}$; $\frac{95}{200}$; $\frac{378}{560}$.

21. Reduce to hundredths: $\frac{1}{4}$; $\frac{3}{5}$; $\frac{9}{10}$; $\frac{11}{20}$; $\frac{14}{25}$; $\frac{120}{400}$; $\frac{252}{900}$.

22. Reduce to fractions having the denominator 72: $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{5}{9}$; $\frac{11}{12}$; $\frac{19}{24}$; $\frac{25}{36}$; $\frac{98}{144}$; $\frac{188}{288}$; $\frac{412}{648}$.

23. Reduce $\frac{2}{3}$ and $\frac{9}{16}$ each to a fraction whose denominator is 48; 96; 144; 240; 384; 528.

112. Reduction of integers or mixed numbers to fractions.

1. How many half dollars are there in one dollar? in two dollars? in \$3? in \$$3\frac{1}{2}$?

2. How many quarters are there in \$1? in \$2? in \$$2\frac{1}{4}$?

3. How many fourths are there in 3? in $3\frac{1}{4}$? in $3\frac{3}{4}$?

4. Express as thirds: 1; $2\frac{1}{3}$; $3\frac{2}{3}$; 8; $6\frac{1}{3}$; 9.

5. Reduce to fifths: 1; 3; $2\frac{1}{5}$; $4\frac{2}{5}$; 6; $5\frac{3}{5}$; $7\frac{4}{5}$.

113. A number expressed by an integer and a fraction is called a **mixed number.**

EXERCISES

114. 1. Reduce to sixths: 4; 5; $2\frac{1}{6}$; $3\frac{5}{6}$; 7; 9; 12.

2. Reduce to eighths: 3; $2\frac{1}{8}$; 7; $4\frac{3}{8}$; 9; $6\frac{5}{8}$; 11.

3. Reduce to tenths: 6; $8\frac{3}{10}$; 9; $2\frac{7}{10}$; 5; 8; 10.

Reduce to a fraction:

4. $3\frac{1}{2}$	7. $5\frac{1}{6}$	10. $9\frac{4}{5}$	13. $3\frac{3}{8}$	16. $10\frac{1}{3}$
5. $6\frac{2}{3}$	8. $7\frac{2}{5}$	11. $4\frac{5}{6}$	14. $2\frac{5}{9}$	17. $11\frac{1}{2}$
6. $8\frac{3}{4}$	9. $4\frac{1}{8}$	12. $6\frac{3}{7}$	15. $4\frac{7}{12}$	18. $12\frac{3}{5}$

WRITTEN EXERCISES

115. 1. Reduce $24\frac{4}{5}$ to fifths.

$$24 = \frac{120}{5}$$

$$\frac{120}{5} + \frac{4}{5} = \frac{124}{5}$$

How many fifths are there in 1? in 24? How many fifths are 120 fifths and 4 fifths, that is, $\frac{120}{5}$ and $\frac{4}{5}$?

Then how many fifths are $24\frac{4}{5}$?

Reduce to a fraction:

2. $23\frac{1}{2}$	6. $28\frac{1}{8}$	10. $52\frac{3}{5}$	14. $48\frac{5}{12}$	18. $142\frac{3}{4}$
3. $14\frac{1}{4}$	7. $44\frac{2}{5}$	11. $81\frac{5}{6}$	15. $65\frac{7}{15}$	19. $375\frac{4}{5}$
4. $37\frac{1}{3}$	8. $75\frac{3}{4}$	12. $93\frac{7}{8}$	16. $72\frac{9}{16}$	20. $561\frac{5}{6}$
5. $16\frac{1}{6}$	9. $63\frac{2}{3}$	13. $78\frac{3}{10}$	17. $86\frac{17}{24}$	21. $826\frac{7}{8}$

116. Reduction of improper fractions to integers or mixed numbers.

1. How many quarter dollars are equal to \$1? to \$2?

To how many dollars are 8 quarters equal? 12 quarters? $\$\frac{8}{4}$? $\$\frac{12}{4}$? $\$\frac{16}{4}$?

2. How many ones and how many thirds over are $\frac{7}{3}$? $\frac{10}{3}$? $\frac{14}{3}$? $\frac{16}{3}$? $\frac{20}{3}$?

3. Reduce to a mixed number: $\frac{9}{2}$; $\frac{13}{3}$; $\frac{17}{4}$; $\frac{19}{4}$; $\frac{21}{5}$.

4. Find the value of: $\$\frac{6}{2}$; $\$\frac{7}{2}$; $\$\frac{9}{4}$; $\$\frac{15}{4}$; $\frac{20}{5}$; $\frac{24}{5}$.

117. A fraction *indicates division*, and its **value** is the quotient of the numerator divided by the denominator.

118. A fraction whose numerator is less than its denominator is called a **proper fraction.**

The value of a proper fraction is less than 1.

119. A fraction whose numerator equals or exceeds its denominator is called an **improper fraction.**

The value of an improper fraction is 1 or more than 1.

EXERCISES

120. 1. Change to an integer: $\frac{9}{3}$; $\frac{10}{2}$; $\frac{10}{5}$; $\frac{12}{4}$; $\frac{12}{6}$; $\frac{18}{6}$; $\frac{20}{4}$; $\frac{20}{5}$; $\frac{24}{8}$; $\frac{24}{12}$.

2. Express as a mixed number: $\frac{5}{2}$; $\frac{11}{5}$; $\frac{13}{4}$; $\frac{11}{3}$; $\frac{13}{6}$; $\frac{15}{2}$; $\frac{16}{3}$; $\frac{17}{6}$; $\frac{21}{4}$; $\frac{23}{5}$; $\frac{25}{4}$.

3. Reduce to an integer or a mixed number: $\frac{14}{2}$; $\frac{17}{3}$; $\frac{24}{4}$; $\frac{26}{5}$; $\frac{35}{5}$; $\frac{39}{4}$; $\frac{24}{6}$; $\frac{40}{8}$; $\frac{27}{8}$; $\frac{29}{6}$.

WRITTEN EXERCISES

121. 1. Reduce $\frac{94}{4}$ to a mixed number.

$\frac{94}{4} = 94 \div 4 = 23\frac{2}{4} = 23\frac{1}{2}$

How many fourths are equal to 1? How many times does 94 contain 4? How many ones, then, are there in 94 fourths? how many fourths over? Express $\frac{2}{4}$ in its lowest terms. Then $\frac{94}{4} = 23\frac{1}{2}$.

Reduce to an integer or a mixed number:

2. $\frac{77}{3}$	5. $\frac{125}{6}$	8. $\frac{226}{12}$	11. $\frac{608}{32}$	14. $\frac{2000}{125}$
3. $\frac{98}{5}$	6. $\frac{294}{8}$	9. $\frac{462}{16}$	12. $\frac{925}{75}$	15. $\frac{2844}{144}$
4. $\frac{84}{4}$	7. $\frac{343}{7}$	10. $\frac{500}{24}$	13. $\frac{876}{96}$	16. $\frac{6000}{256}$

122. Reduction to least common denominator.

1. Change $\frac{1}{2}$ and $\frac{2}{3}$ each to sixths; to twelfths; to some other *common* denominator.

Which common denominator is the smallest, or *least?*

2. What is the least number that will exactly contain the denominator of each of the fractions $\frac{1}{3}$ and $\frac{3}{4}$?

Then, what is the least common denominator to which these fractions can be reduced?

Reduce $\frac{1}{3}$ and $\frac{3}{4}$ each to twelfths.

123. Fractions that have the same denominator are said to have a **common denominator**, and are called **similar fractions.**

124. Similar fractions that have the smallest common denominator possible are said to have the **least common denominator.**

EXERCISES

125. Reduce to similar fractions:

1. $\frac{1}{2}$ and $\frac{1}{6}$	4. $\frac{5}{12}$ and $\frac{1}{2}$	7. $\frac{2}{3}$ and $\frac{5}{6}$	10. $\frac{3}{4}$ and $\frac{5}{12}$
2. $\frac{3}{8}$ and $\frac{1}{2}$	5. $\frac{1}{4}$ and $\frac{5}{8}$	8. $\frac{1}{3}$ and $\frac{4}{9}$	11. $\frac{4}{5}$ and $\frac{9}{10}$
3. $\frac{1}{2}$ and $\frac{3}{10}$	6. $\frac{1}{12}$ and $\frac{1}{4}$	9. $\frac{7}{12}$ and $\frac{2}{3}$	12. $\frac{11}{12}$ and $\frac{5}{6}$

Reduce to fractions having the least common denominator:

13. $\frac{1}{2}$ and $\frac{3}{5}$	**15.** $\frac{1}{2}$, $\frac{5}{6}$, $\frac{1}{12}$	**17.** $\frac{2}{5}$, $\frac{1}{2}$, $\frac{9}{10}$
14. $\frac{1}{4}$ and $\frac{1}{6}$	**16.** $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{12}$	**18.** $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{12}$

WRITTEN EXERCISES

126. **1.** Reduce $\frac{5}{8}$ and $\frac{7}{12}$ to similar fractions.

$\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$

$\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$

To change these fractions to similar fractions, a number must be selected for a common denominator that will exactly contain each of the given denominators. It is always desirable to select the smallest number that will contain them.

It is seen that 24 will contain 8 and 12, and that no smaller number will contain both without a remainder.

Since $24 \div 8 = 3$, the terms of $\frac{5}{8}$ must be multiplied by 3, and since $24 \div 12 = 2$, the terms of $\frac{7}{12}$ must be multiplied by 2, giving for results the similar fractions $\frac{15}{24}$ and $\frac{14}{24}$.

Reduce to similar fractions:

2. $\frac{3}{4}$ and $\frac{1}{5}$	**6.** $\frac{5}{8}$, $\frac{1}{4}$, $\frac{7}{16}$	**10.** $\frac{1}{2}$, $\frac{3}{5}$, $\frac{1}{6}$
3. $\frac{1}{6}$ and $\frac{3}{8}$	**7.** $\frac{1}{3}$, $\frac{7}{8}$, $\frac{1}{4}$	**11.** $\frac{3}{4}$, $\frac{2}{9}$, $\frac{5}{12}$
4. $\frac{5}{9}$ and $\frac{5}{6}$	**8.** $\frac{4}{5}$, $\frac{2}{3}$, $\frac{9}{10}$	**12.** $\frac{2}{5}$, $\frac{1}{8}$, $\frac{11}{20}$
5. $\frac{7}{8}$ and $\frac{8}{7}$	**9.** $\frac{3}{4}$, $\frac{5}{8}$, $\frac{4}{5}$	**13.** $\frac{7}{8}$, $\frac{7}{32}$, $\frac{9}{16}$

127. It is not always easy to discover by inspection the *smallest* number that will exactly contain each of the given denominators, that is, the *least common denominator* (l. c. d.).

The l. c. d. may be found, however, by factoring the denominators, for it is the product of all their *different prime* factors, each factor used the greatest number of times that it occurs in any denominator.

Thus, if the given denominators are 6, 4, and 16, factoring we find: $6 = 2 \times 3$; $4 = 2 \times 2$; and $16 = 2 \times 2 \times 2 \times 2$.

Then, the factors of the l. c. d. are 2, 2, 2, 2 (the greatest number of 2's found in any denominator), and 3 (the only factor of any of them not already taken).

Hence, the l. c. d. $= 2 \times 2 \times 2 \times 2 \times 3 = 48$.

WRITTEN EXERCISES

1. Reduce $\frac{7}{12}$, $\frac{5}{8}$, and $\frac{9}{10}$ to fractions having the l. c. d.

Factoring the denominators, $12 = 2 \times 2 \times 3$; $8 = 2 \times 2 \times 2$; and $10 = 2 \times 5$.

The factors of the l. c. d., then, are 2, 2, 2, 3, and 5.

Hence, the l. c. d. $= 2 \times 2 \times 2 \times 3 \times 5 = 120$, and the fractions become $\frac{70}{120}$, $\frac{75}{120}$, and $\frac{108}{120}$.

Reduce to fractions having the l. c. d. :

2. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{9}{20}$

3. $\frac{2}{5}$, $\frac{14}{25}$, $\frac{8}{15}$

4. $\frac{7}{8}$, $\frac{9}{14}$, $\frac{43}{56}$

5. $\frac{4}{15}$, $\frac{23}{30}$, $\frac{17}{20}$

6. $\frac{7}{12}$, $\frac{11}{24}$, $\frac{13}{18}$

7. $\frac{5}{36}$, $\frac{15}{16}$, $\frac{19}{24}$

8. $\frac{3}{8}$, $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{32}$

9. $\frac{1}{4}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{6}$

10. $\frac{5}{12}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{5}{36}$, $\frac{3}{4}$

Addition and Subtraction of Fractions

128. **1.** How many fifths are $\frac{3}{5} + \frac{2}{5}$? how many ones?

2. How many fourths are $\frac{3}{4} - \frac{1}{4}$? how many halves?

3. How many tenths are $\frac{9}{10} + \frac{7}{10}$? how many ones and how many tenths over? how many fifths over?

4. How many eighths are there in $\frac{1}{2} + \frac{3}{8}$? in $\frac{1}{2} - \frac{3}{8}$? in $\frac{5}{8} + \frac{1}{4}$? in $\frac{5}{8} - \frac{1}{4}$? in $\frac{3}{4} + \frac{3}{8}$? in $\frac{3}{4} - \frac{3}{8}$?

5. How much is $\frac{5}{6} + \frac{2}{3}$? $\frac{5}{6} - \frac{2}{3}$? $\frac{1}{2} + \frac{1}{3}$? $\frac{1}{2} - \frac{1}{3}$? $\frac{3}{4} + \frac{2}{3}$? $\frac{3}{4} - \frac{2}{3}$? $\frac{1}{4} + \frac{1}{6}$? $\frac{1}{4} - \frac{1}{6}$?

129. *Fractions must be made similar before they can be added or subtracted.*

EXERCISES

130. Add or subtract as the signs indicate :

1. $\frac{1}{4} + \frac{1}{2}$

2. $\frac{1}{2} - \frac{1}{6}$

3. $\frac{1}{8} + \frac{1}{2}$

4. $\frac{1}{3} - \frac{1}{6}$

5. $\frac{2}{9} + \frac{2}{3}$

6. $\frac{3}{4} - \frac{5}{8}$

7. $\frac{7}{10} + \frac{1}{2}$

8. $\frac{1}{2} - \frac{5}{12}$

9. $\frac{1}{12} + \frac{2}{3}$

10. $\frac{3}{4} - \frac{7}{12}$

11. $\frac{2}{5} + \frac{3}{10}$

12. $\frac{11}{12} - \frac{5}{6}$

13. $\frac{1}{2} + \frac{2}{3}$

14. $\frac{4}{5} - \frac{1}{2}$

15. $\frac{1}{4} + \frac{1}{3}$

16. $\frac{5}{6} - \frac{3}{4}$

First add; then subtract:

17. $4\frac{3}{4}$	**18.** $7\frac{5}{8}$	**19.** $6\frac{1}{2}$	**20.** $5\frac{7}{12}$	**21.** $8\frac{1}{4}$
$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{10}$	$1\frac{1}{3}$	$4\frac{1}{12}$
22. $9\frac{5}{6}$	**23.** $3\frac{3}{4}$	**24.** $5\frac{11}{12}$	**25.** $7\frac{4}{5}$	**26.** $6\frac{7}{12}$
$3\frac{2}{3}$	$1\frac{3}{8}$	$2\frac{1}{2}$	$4\frac{3}{10}$	$3\frac{1}{6}$
27. $7\frac{1}{2}$	**28.** $8\frac{3}{5}$	**29.** $10\frac{3}{4}$	**30.** $12\frac{5}{6}$	**31.** $9\frac{9}{10}$
$5\frac{1}{3}$	$3\frac{1}{2}$	$6\frac{1}{3}$	$7\frac{1}{4}$	$4\frac{2}{5}$

32. A newsboy earned $\$\frac{3}{4}$ and gave his mother $\$\frac{1}{2}$. What part of a dollar did he have left?

33. How wide must a piece of silk be cut to make a band $\frac{1}{2}$ in. wide with $\frac{1}{8}$ in. turned in on each side?

34. Mary spent $\frac{1}{8}$ and $\frac{1}{4}$ of her money. What part of it had she left?

35. Find the perimeter of an envelope 6 in. by $3\frac{1}{4}$ in.

36. A $3\frac{1}{2}$-inch nail is driven through a $1\frac{1}{4}$-inch board into a post. How far does it extend into the post?

WRITTEN EXERCISES

131. **1.** Find the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$.

$\frac{5}{6} + \frac{7}{8} + \frac{11}{12} =$

$\frac{20}{24} + \frac{21}{24} + \frac{22}{24} = \frac{63}{24} = 2\frac{5}{8}$

You have learned that the fractions must be changed to similar fractions before they can be added. The l.c.d. of the given fractions is 24; then, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, and $\frac{11}{12} = \frac{22}{24}$. Adding these similar fractions, the sum is found to be $\frac{63}{24} = 2\frac{15}{24}$ or $2\frac{5}{8}$.

Add:

2. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{12}$	**6.** $\frac{3}{4}$, $\frac{4}{5}$, $\frac{7}{10}$	**10.** $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{5}{6}$
3. $\frac{1}{4}$, $\frac{5}{6}$, $\frac{7}{12}$	**7.** $\frac{5}{6}$, $\frac{2}{3}$, $\frac{9}{16}$	**11.** $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{6}$, $\frac{1}{3}$
4. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{11}{15}$	**8.** $\frac{8}{9}$, $\frac{3}{4}$, $\frac{17}{18}$	**12.** $\frac{2}{3}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{3}{4}$
5. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{13}{16}$	**9.** $\frac{7}{8}$, $\frac{5}{16}$, $\frac{11}{12}$	**13.** $\frac{5}{9}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{71}{72}$

14. Add $23\frac{1}{2}$, $49\frac{4}{5}$, and $36\frac{3}{4}$.

$23\frac{1}{2} = 23\frac{10}{20}$
$49\frac{4}{5} = 49\frac{16}{20}$
$36\frac{3}{4} = 36\frac{15}{20}$
$110\frac{1}{20}$

Reducing the fractions to fractions having the l. c. d., they become $\frac{10}{20}$, $\frac{16}{20}$, and $\frac{15}{20}$, whose sum is $\frac{41}{20}$ or $2\frac{1}{20}$. Writing $\frac{1}{20}$ under the fractions and adding the 2 to the integers, the whole sum is found to be $110\frac{1}{20}$.

Add:

15. $19\frac{1}{3}$	**16.** $25\frac{1}{5}$	**17.** $44\frac{1}{2}$	**18.** $37\frac{1}{6}$	**19.** $56\frac{2}{3}$
$28\frac{1}{2}$	$38\frac{5}{8}$	$68\frac{2}{3}$	$91\frac{1}{2}$	$75\frac{3}{4}$
$32\frac{1}{6}$	$74\frac{1}{2}$	$47\frac{3}{4}$	$84\frac{1}{4}$	$94\frac{5}{6}$
20. $24\frac{1}{2}$	**21.** $41\frac{1}{3}$	**22.** $96\frac{1}{4}$	**23.** $77\frac{2}{3}$	**24.** $48\frac{7}{10}$
$85\frac{3}{4}$	$9\frac{1}{2}$	$74\frac{5}{6}$	$13\frac{3}{4}$	$64\frac{7}{12}$
$62\frac{1}{3}$	$58\frac{1}{6}$	$26\frac{2}{3}$	$5\frac{7}{8}$	$92\frac{7}{15}$
$70\frac{1}{6}$	$37\frac{3}{5}$	$42\frac{3}{8}$	$84\frac{1}{10}$	$58\frac{7}{20}$

25. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

$\frac{7}{8} - \frac{5}{12} =$
$\frac{21}{24} - \frac{10}{24} = \frac{11}{24}$

We have learned that the fractions must be changed to similar fractions before one can be subtracted from the other. Reducing them to their least common denominator, $\frac{7}{8}$ becomes $\frac{21}{24}$ and $\frac{5}{12}$ becomes $\frac{10}{24}$; then, the difference between $\frac{21}{24}$ and $\frac{10}{24}$ is seen to be $\frac{11}{24}$.

Find the difference between:

26. $\frac{3}{4}$ and $\frac{5}{16}$	**29.** $\frac{4}{5}$ and $\frac{1}{4}$	**32.** $\frac{14}{15}$ and $\frac{1}{2}$	**35.** $\frac{15}{16}$ and $\frac{5}{12}$
27. $\frac{19}{20}$ and $\frac{2}{5}$	**30.** $\frac{3}{8}$ and $\frac{1}{6}$	**33.** $\frac{11}{12}$ and $\frac{1}{8}$	**36.** $\frac{17}{20}$ and $\frac{4}{15}$
28. $\frac{7}{8}$ and $\frac{11}{24}$	**31.** $\frac{5}{6}$ and $\frac{3}{5}$	**34.** $\frac{9}{20}$ and $\frac{1}{4}$	**37.** $\frac{7}{12}$ and $\frac{7}{30}$

38. From $84\frac{3}{8}$ subtract $27\frac{3}{4}$.

$84\frac{3}{8} = 84\frac{3}{8} = 83\frac{11}{8}$
$27\frac{3}{4} = 27\frac{6}{8} = 27\frac{6}{8}$
$56\frac{5}{8}$

Reducing the fractions to similar fractions, it is seen that $\frac{6}{8}$ cannot be subtracted from $\frac{3}{8}$; hence, 1 is taken from 84, changed to eighths, and combined with $\frac{3}{8}$; then $\frac{6}{8}$ is subtracted from $\frac{11}{8}$ and 27 from 83, giving the remainder $56\frac{5}{8}$.

Subtract:

39. $35\frac{1}{2}$	**43.** $42\frac{3}{10}$	**47.** $77\frac{1}{3}$	**51.** $54\frac{7}{8}$	**55.** $275\frac{3}{8}$
$19\frac{3}{4}$	$26\frac{4}{5}$	$49\frac{1}{2}$	$25\frac{1}{6}$	$184\frac{13}{24}$
40. $86\frac{1}{6}$	**44.** $68\frac{5}{6}$	**48.** $95\frac{1}{2}$	**52.** $36\frac{1}{4}$	**56.** $347\frac{7}{9}$
$22\frac{1}{2}$	$35\frac{5}{12}$	$54\frac{2}{5}$	$17\frac{3}{5}$	$159\frac{19}{36}$
41. $57\frac{1}{4}$	**45.** $97\frac{1}{3}$	**49.** $38\frac{2}{3}$	**53.** $71\frac{1}{3}$	**57.** $742\frac{14}{25}$
$28\frac{5}{8}$	$42\frac{7}{12}$	$15\frac{3}{4}$	$43\frac{3}{8}$	$475\frac{33}{50}$
42. $74\frac{2}{9}$	**46.** $59\frac{3}{4}$	**50.** $82\frac{1}{4}$	**54.** $96\frac{3}{7}$	**58.** $503\frac{7}{20}$
$39\frac{2}{3}$	$21\frac{11}{12}$	$64\frac{5}{6}$	$78\frac{3}{4}$	$324\frac{5}{12}$

WRITTEN EXERCISES

132. **1.** John could run a hundred yards in $14\frac{1}{5}$ seconds and George in $13\frac{4}{5}$ seconds. How much more quickly could George run the distance than John?

2. A wagon with its load weighs $2\frac{1}{4}$ tons. The wagon alone weighs $\frac{3}{5}$ of a ton. Find the weight of the load.

3. A man left $\frac{1}{3}$ of his property to his wife, $\frac{3}{8}$ of it to his daughters, and the rest to his son. What part of the property did the son receive?

4. Add seventy-five and seven eighths, forty-three and five sixths, and ninety-one and five twelfths.

5. Mr. Brown burned $12\frac{3}{4}$ tons of coal in his furnace last winter, $11\frac{13}{20}$ tons the winter before, and $11\frac{1}{5}$ tons the winter before that. How much coal did he use in three winters?

6. From the sum of thirty-nine and four fifths and sixty-two and three eighths subtract their difference.

7. A miller shoveled $38\frac{1}{2}$ bushels of corn from a bin containing $75\frac{3}{8}$ bushels into a bin containing $27\frac{3}{4}$ bushels. How many bushels of corn were there then in each bin?

8. A farmer drew his potatoes to market in four loads, containing, respectively, $60\frac{5}{6}$ bu., $57\frac{3}{4}$ bu., $62\frac{1}{2}$ bu., and $54\frac{7}{12}$ bu. How many bushels did he take to market?

9. If it costs on the average $4\frac{9}{20}$¢ per pound to raise cotton and get it ready for market, what is the profit per pound when it sells at $8\frac{7}{8}$¢? at $9\frac{3}{4}$¢? at $10\frac{3}{16}$¢? at $12\frac{1}{2}$¢? at $13\frac{1}{4}$¢?

Multiplication of Fractions

133. Multiplication of integers by fractions.

1. How many are $\frac{1}{3}$ of 9? $\frac{2}{3}$ of 9? $\frac{1}{4}$ of 12? $\frac{3}{4}$ of 12?

2. Find $\frac{2}{5}$ of 10; $\frac{1}{5}$ of 20. How does $\frac{2}{5}$ of 10 compare with $\frac{1}{5}$ of 20? 2 times $\frac{1}{5}$ of 10 with $\frac{1}{5}$ of 2 times 10?
Tell two ways of finding $\frac{2}{5}$ of 10.

3. Find $\frac{2}{5}$ of 11 by finding $\frac{1}{5}$ of 2 times 11; $\frac{3}{4}$ of 9 by finding $\frac{1}{4}$ of 3 times 9.

4. Find $\frac{2}{3}$ of 8; $\frac{2}{3}$ of 9; $\frac{3}{4}$ of 8; $\frac{3}{4}$ of 9; $\frac{4}{5}$ of 10; $\frac{5}{6}$ of 10.

134. Finding a fractional part of a number is called **multiplying by a fraction.**

WRITTEN EXERCISES

135. 1. Find $\frac{7}{8}$ of 16; $\frac{7}{8}$ of 18.

$\frac{1}{8}$ of $16 = 16 \div 8 = 2$; $\frac{7}{8}$ of $18 = \frac{1}{8}$ of $7 \times 18 = \frac{1}{8}$ of 126;

$\frac{7}{8}$ of $16 = 7$ times $2 = 14$. $\frac{1}{8}$ of $126 = 126 \div 8 = 15\frac{6}{8} = 15\frac{3}{4}$.

Since 8 is exactly contained in 16, in finding $\frac{7}{8}$ of 16 it is easier and shorter first to *divide* the integer 16 by the *denominator* 8 and then *multiply* the quotient by the *numerator* 7, obtaining the result 14.

Since 8 is *not* exactly contained in 18, in finding $\frac{7}{8}$ of 18 it is easier and shorter first to *multiply* the integer 18 by the *numerator* 7 and then *divide* the product by the *denominator* 8.

It is still shorter to indicate the work and cancel.

$$\frac{7 \times \overset{9}{\cancel{18}}}{\underset{4}{\cancel{8}}} = \frac{63}{4} = 15\frac{3}{4}.$$

Find, in the shortest way:

2. $\frac{3}{4}$ of 36	6. $\frac{3}{5}$ of 75	10. $\frac{3}{8}$ of 46	14. $\frac{2}{9}$ of 81
3. $\frac{3}{4}$ of 38	7. $\frac{4}{5}$ of 89	11. $\frac{4}{7}$ of 63	15. $\frac{7}{8}$ of 98
4. $\frac{2}{3}$ of 96	8. $\frac{5}{6}$ of 93	12. $\frac{5}{8}$ of 77	16. $\frac{3}{10}$ of 85
5. $\frac{2}{3}$ of 97	9. $\frac{3}{4}$ of 81	13. $\frac{2}{5}$ of 59	17. $\frac{5}{12}$ of 92

Multiply, using cancellation where possible:

18. 48 by $\frac{5}{6}$	23. 146 by $\frac{2}{3}$	28. 295 by $\frac{7}{10}$
19. 50 by $\frac{5}{6}$	24. 421 by $\frac{3}{4}$	29. 507 by $\frac{5}{12}$
20. 84 by $\frac{3}{8}$	25. 937 by $\frac{2}{5}$	30. 348 by $\frac{15}{16}$
21. 68 by $\frac{4}{5}$	26. 742 by $\frac{5}{8}$	31. 864 by $\frac{13}{20}$
22. 76 by $\frac{3}{4}$	27. 894 by $\frac{5}{6}$	32. 928 by $\frac{17}{24}$

33. Multiply 285 by $27\frac{5}{6}$.

285
$27\frac{5}{6}$
6)1425
$237\frac{1}{2}$, product by $\frac{5}{6}$
1995, product by 7
570, product by 20
$7932\frac{1}{2}$, product by $27\frac{5}{6}$

Multiplying 285 by the numerator 5 and dividing the quotient by 6, $237\frac{1}{2}$ is found to be the product by $\frac{5}{6}$.

Multiplying by 7 and by 20 (2 tens), writing these partial products in their proper places as in the process, and then adding, the product of 285 multiplied by $27\frac{5}{6}$ is found to be $7932\frac{1}{2}$.

Multiply:

34. 37 by $17\frac{3}{4}$	41. 134 by $36\frac{1}{2}$	48. 346 by $125\frac{2}{3}$
35. 46 by $25\frac{2}{3}$	42. 341 by $91\frac{1}{4}$	49. 517 by $206\frac{3}{4}$
36. 54 by $42\frac{3}{5}$	43. 604 by $43\frac{2}{3}$	50. 628 by $409\frac{4}{5}$
37. 79 by $38\frac{1}{6}$	44. 467 by $56\frac{3}{4}$	51. 497 by $342\frac{1}{6}$
38. 294 by $18\frac{1}{5}$	45. 528 by $87\frac{2}{5}$	52. 836 by $570\frac{5}{8}$
39. 723 by $26\frac{3}{8}$	46. 279 by $68\frac{5}{6}$	53. 288 by $735\frac{7}{10}$
40. 618 by $17\frac{5}{6}$	47. 842 by $74\frac{7}{8}$	54. 969 by $678\frac{11}{12}$

EXERCISES

136. Find the cost of:

1. $\frac{3}{4}$ pound of ginger at 28 cents a pound.
2. $2\frac{1}{2}$ pounds of coffee at 30 cents a pound.
3. $1\frac{1}{4}$ gallons of vinegar at 26 cents a gallon.
4. $3\frac{2}{3}$ dozen bananas at 12 cents a dozen.
5. $3\frac{1}{2}$ pounds of figs at 16 cents a pound.
6. $1\frac{3}{4}$ gallons of molasses at 54 cents a gallon.
7. $4\frac{1}{2}$ pounds of cheese at 11 cents a pound.
8. If I am on a train that goes at the rate of 40 miles an hour, how far do I travel in $1\frac{1}{2}$ hours?
9. How far is it from Hurst to Troy, if John uses up $\frac{3}{5}$ of a thousand-mile railroad book in traveling between them?
10. If it takes Mary 45 minutes to walk to school and Clara $\frac{2}{3}$ as long, how many minutes does it take Clara?
11. A lesson in arithmetic contained 24 exercises. Elmer solved $\frac{5}{6}$ of them. How many did he solve?
12. How many cents are there in $\frac{3}{5}$ of a dollar?
13. A child should sleep at least $\frac{3}{8}$ of the time. How many of the 24 hours in a day should every child sleep?
14. A farmer who had 21 sheep sold $\frac{1}{3}$ of them at \$4 each. How much did he receive for the sheep he sold?
15. Eleanor bought $4\frac{1}{4}$ yards of ribbon at 20 cents a yard. How much did it cost her?
16. Mr. Williams paid 32 cents a bushel for a bag of oats containing $2\frac{1}{8}$ bushels. How much did the oats cost?
17. I bought $1\frac{3}{4}$ pounds of 40-cent tea and handed the grocer a dollar. How much change did he give me?
18. Find the cost of $5\frac{1}{2}$ yards of percale at 12¢ a yard.

WRITTEN EXERCISES

137. Find the cost of:

1. 25 shovels at $\$\frac{3}{4}$ each.

SUGGESTION. — At $\$\frac{3}{4}$ each the cost is $\frac{3}{4}$ of what it would be at \$1 each.

2. 278 barrels of cement at $\$1\frac{1}{4}$ a barrel.

3. $6\frac{2}{5}$ tons of coal at \$6.25 a ton.

4. $37\frac{5}{8}$ acres of land at \$245 an acre.

5. 465 bushels of apples at $\$\frac{7}{10}$ a bushel.

6. $4\frac{5}{8}$ miles of wire fencing at \$174 per mile.

7. 2542 pounds of iron pipe at $8\frac{3}{4}$ cents a pound.

8. 32,864 pounds of cotton at $9\frac{7}{16}$ cents a pound.

9. If it requires $1\frac{3}{4}$ bushels of seed wheat to sow one acre, how many bushels are needed to sow 26 acres?

10. An apple tree yielded $12\frac{3}{8}$ bushels of apples that sold for 72 cents a bushel. Find the income from this tree.

11. A meadow of 96 acres produced on the average $1\frac{4}{5}$ tons of hay per acre. What was the total production?

12. A man rented a house at the rate of \$480 a year. He lived in it $4\frac{5}{12}$ years. How much rent did he pay?

13. A chimney measuring 150 feet from the bottom of its foundation is $\frac{7}{8}$ above ground. How high is it above ground?

14. If Ruth has read $\frac{9}{14}$ of the 336 pages in her book, how many pages has she read?

15. At $\$\frac{9}{10}$ a foot how much will it cost to drill an oil well 2256 feet deep?

16. A flock of 440 sheep yielded on the average $3\frac{15}{16}$ pounds of wool each. Find the total yield.

17. Find the total cost of a crop of coffee from a plantation of 24,000 trees that yield an average of $1\frac{3}{5}$ pounds each, if the cost of raising is $4\frac{2}{3}$ cents per pound.

138. Multiplication of fractions by integers.

1. How many fifths are 3 times 1 fifth? 3 times $\frac{1}{5} = ?$

2. How many eighths are 4 times $\frac{1}{8}$? Express the result in its lowest terms.

You have multiplied $\frac{1}{8}$ by 4, obtaining the product $\frac{1}{2}$. Tell how you did it.

3. Divide the denominator of $\frac{1}{8}$ by 4. How does the result compare with the product found by multiplying $\frac{1}{8}$ by 4?

In what other way, then, may we sometimes multiply a fraction by an integer?

4. Multiply $\frac{1}{4}$ by 2 in two ways; $\frac{5}{6}$ by 3; $\frac{3}{8}$ by 2; $\frac{2}{9}$ by 3; $\frac{5}{12}$ by 6.

5. Multiply $\frac{1}{2}$ by 3; $\frac{2}{3}$ by 4; $\frac{3}{4}$ by 5; $\frac{4}{5}$ by 3; $\frac{7}{8}$ by 5.

6. Find 2 times 4; 2 times $\frac{1}{3}$; 2 times $4\frac{1}{3}$; 3 times $2\frac{1}{2}$; $5 \times 3\frac{1}{3}$; $6 \times 2\frac{1}{4}$; $3 \times 5\frac{2}{3}$.

139. *Multiplying the numerator or dividing the denominator of a fraction by an integer multiplies the fraction by that integer.*

EXERCISES

140. Multiply:

1. $\frac{2}{3}$ by 8	4. $\frac{5}{6}$ by 2	7. $\frac{3}{10}$ by 5	10. $5\frac{1}{2}$ by 4
2. $\frac{2}{5}$ by 4	5. $\frac{7}{8}$ by 4	8. $\frac{7}{12}$ by 4	11. $2\frac{3}{4}$ by 2
3. $\frac{3}{4}$ by 2	6. $\frac{8}{9}$ by 3	9. $\frac{9}{16}$ by 8	12. $7\frac{1}{6}$ by 6

13. At $\$\frac{2}{5}$ a yard, how much will 4 yards of cloth cost?

14. A family uses $\frac{3}{8}$ of a pound of butter at each meal. How much butter is used on the table each day?

15. Dr. Jones pays $\$3\frac{1}{2}$ a month for his telephone. How much does it cost him a year?

16. If a man earns $\$2\frac{3}{4}$ a day, how much does he earn in a week of 6 working days?

17. How far can James walk in 5 hours, walking $3\frac{2}{3}$ miles per hour?

18. Mr. Seymour bought a set of 6 books for his library. How much did they cost, if the price was $\$1\frac{3}{4}$ each?

19. George earned $\$2\frac{7}{10}$ a week delivering papers. How much did he earn in 4 weeks?

20. Find the cost of 5 tons of coal at $\$6\frac{1}{4}$ a ton and 3 barrels of flour at $\$5\frac{4}{5}$ a barrel.

21. If the cost of 4 yards of ribbon is $\$1\frac{2}{5}$, what is the cost of 12 yards of the same kind?

SOLUTION. — 12 is 3 times 4; then, the cost of 12 yards is 3 times the cost of 4 yards; that is, the cost of 12 yards is 3 times $\$1\frac{2}{5}$ or $\$4\frac{1}{5}$.

22. If a girl in a factory earns $\$1\frac{3}{4}$ for making 4 shirtwaists, how much will she earn for making 12?

23. If she makes 3 shirtwaists in $10\frac{1}{2}$ hours, how long will it take her to make 12?

24. A man paid $\$3\frac{1}{2}$ for the use of a boat for 7 days. At that rate how much would it cost to hire it for 21 days?

25. A farmer sold 6 bushels of oats for $\$2\frac{1}{10}$. At that rate how much should he receive for 48 bushels?

26. A fruit dealer sold 4 dozen oranges for $\$1\frac{4}{5}$. How much should he receive for 16 dozen at the same rate?

WRITTEN EXERCISES

141. 1. Find 8 times $\frac{13}{24}$.

$$8 \times \frac{13}{24} = \frac{8 \times 13}{24} = \frac{13}{3} = 4\frac{1}{3}$$

We have learned that to find 8 times $\frac{13}{24}$ we may multiply the numerator 13 by 8. Indicating this as in the process, canceling, and reducing, the result is found to be $4\frac{1}{3}$.

Find:

2. $5 \times \frac{11}{25}$
3. $6 \times \frac{15}{16}$
4. $8 \times \frac{13}{18}$
5. $7 \times \frac{17}{28}$
6. $4 \times \frac{21}{32}$
7. $9 \times \frac{38}{60}$
8. $7 \times \frac{33}{42}$
9. $8 \times \frac{65}{72}$

Multiply:

10. $\frac{9}{10}$ by 85
11. $\frac{8}{21}$ by 49
12. $\frac{24}{35}$ by 28
13. $\frac{25}{96}$ by 72
14. $\frac{7}{88}$ by 33
15. $\frac{19}{100}$ by 75
16. $\frac{66}{132}$ by 96
17. $\frac{87}{200}$ by 80

18. Multiply $45\frac{3}{8}$ by 26.

$$\begin{array}{r} 45\frac{3}{8} \\ 26 \\ \hline 9\frac{3}{4} \\ 270 \\ 90 \\ \hline 1179\frac{3}{4} \end{array}$$

Multiplying the fraction $\frac{3}{8}$ by 26, the product is found to be $9\frac{3}{4}$.

Multiplying the integer 45 by 6 and by 20 (2 tens), writing these partial products in their proper places and adding, the product of $45\frac{3}{8}$ multiplied by 26 is found to be $1179\frac{3}{4}$.

Multiply:

19. $68\frac{2}{3}$ by 14
20. $94\frac{3}{5}$ by 57
21. $247\frac{3}{4}$ by 6
22. $468\frac{5}{6}$ by 9
23. $306\frac{1}{2}$ by 27
24. $184\frac{7}{8}$ by 78
25. $536\frac{1}{6}$ by 49
26. $845\frac{4}{5}$ by 96
27. $628\frac{5}{8}$ by 124
28. $459\frac{3}{4}$ by 205
29. $738\frac{7}{10}$ by 462
30. $906\frac{5}{12}$ by 369

WRITTEN EXERCISES

142. Find the cost of:

1. 5 pounds of butter at $23\frac{3}{4}$ cents a pound.
2. 8 cans of tomatoes at $16\frac{2}{3}$ cents a can.
3. 2 turkeys weighing $14\frac{1}{2}$ and $16\frac{3}{4}$ pounds at 24 cents a pound.
4. $18\frac{3}{4}$ yards of velvet at \$3 a yard.
5. $24\frac{5}{6}$ dozen buttons at 36 cents a dozen.
6. $85\frac{7}{8}$ yards of matting at 42 cents a yard.
7. A cubic foot of water weighs $62\frac{1}{2}$ pounds. Find the weight of 25 cubic feet of water.
8. If a steamer burns $457\frac{3}{5}$ tons of coal a day, how many tons will it burn on a voyage of 7 days?

9. How far will a train run in 18 hours at the rate of $53\frac{3}{8}$ miles an hour?

10. When potatoes are selling so that a man gets $\$37\frac{1}{2}$ for a load of 50 bushels, how much is his crop of 450 bushels worth?

11. Mr. Joy has 12 bins in his granary, and they hold $192\frac{7}{8}$ bushels each. How many bushels of grain do they all hold?

12. If the multiplicand is forty-two and five twelfths, and the multiplier thirty-six, what is the product?

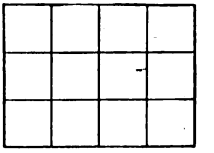
13. A farmer had 6 stacks of hay that contained on the average $16\frac{5}{8}$ tons each. How much was the hay worth at \$14 a ton?

14. Mr. Holmes had a farm of $125\frac{3}{4}$ acres, and he bought an adjoining lot of $86\frac{4}{5}$ acres. How much was the whole farm then worth at \$75 an acre?

Division of Fractions

143. Finding what part one number is of another.

1. How many horizontal rows of squares are there in this oblong? What part of the oblong is one row?



How many squares are there in the oblong? in one row? Then what part of 12 is 4?

How does this result compare with $4 \div 12$, or with $\frac{4}{12}$ reduced to its lowest terms?

Then how may we find what fractional part 4 is of 12 without looking at the diagram?

2. What part of 12 is 8? 3? 6? 9? 10?

3. What part of 16 is 8? 4? 2? What part of 20 is 4? 8? 12? 5? 10? 15?

144. To find what part the second of two numbers is of the first, *the second is divided by the first.*

EXERCISES

145. What part of

1. 18 is 6?	**4.** 15 is 10?	**7.** 24 is 12?
2. 24 is 8?	**5.** 16 is 12?	**8.** 30 is 20?
3. 30 is 5?	**6.** 18 is 15?	**9.** 36 is 18?

10. A boy who had 20 problems to work solved 10 of them. What part of his problems did he solve?

11. A man had 10 miles to drive. When he had driven 8 miles, what part of his journey had he completed?

12. What part of 12 is 4? If 12 spools of thread cost 60 cents, what part of 60 cents will 4 spools cost? how much will 4 spools cost?

13. What part of 15 is 6? If 15 apples cost 25 cents, how much will 6 apples cost?

14. Find the cost of 20 marbles, if 30 marbles cost 12 cents.

15. What part of 75 is 25? If 6 dozen bananas can be bought for 75 cents, what part of 6 dozen can be bought for 25 cents? how many dozen bananas?

16. What part of 50 is 30? If 10 bottles of ink cost 50 cents, how many can be bought for 30 cents?

17. Find how many sheep can be bought for $12, if 9 sheep cost $36.

WRITTEN EXERCISES

146. What part of

1. 48 is 36?	**4.** 120 is 90?	**7.** 250 is 100?
2. 64 is 16?	**5.** 144 is 84?	**8.** 432 is 288?
3. 96 is 72?	**6.** 225 is 75?	**9.** 768 is 336?

10. A farmer raised 180 bushels of oats and sold 120 bushels. What part of his crop did he sell?

11. A grocer bought 400 dozen eggs. When he had sold 144 dozen, what part of them had he sold?

12. Of the 375 bushels of potatoes that Mr. Avery raised 125 bushels decayed. What part of his crop decayed?

13. What part of 64 is 40? I sold 64 head of cattle for $1472. At this rate how much were 40 head worth?

14. If a train runs 860 miles in 20 hours, how far at the same rate will it run in 12 hours?

15. When 28 acres of land sell for $2688, how much are 21 acres worth at the same rate?

16. If a man earns $3570 in 42 months, how much does he earn in 12 months?

17. Mr. Doty sold 64 tons of hay for $1004.80. At that rate, find the value of the rest of his crop, 24 tons.

147. Division of fractions by integers.

1. If $\frac{4}{5}$ of an orange is divided equally between 2 girls, what part of the orange will each girl have? $\frac{4}{5} \div 2 = ?$

In dividing $\frac{4}{5}$ by 2, which term of the fraction is divided?

2. Multiply the denominator of $\frac{4}{5}$ by 2. Express the resulting fraction in its lowest terms. Compare this result with the quotient of $\frac{4}{5}$ divided by 2.

3. In what two ways, then, can $\frac{4}{5}$ be divided by 2?

4. Divide $\frac{3}{4}$ by 3 in two ways; $\frac{4}{6}$ by 2; $\frac{6}{8}$ by 3; $\frac{8}{9}$ by 4; $\frac{10}{12}$ by 5.

5. Find $\frac{2}{3} \div 4$; $\frac{4}{5} \div 6$; $\frac{9}{10} \div 3$; $\frac{7}{8} \div 2$.

6. How many fourths are $2\frac{3}{4}$? Divide $\frac{11}{4}$ by 3. Then how may we divide $2\frac{3}{4}$ by 3?

7. Divide $1\frac{2}{3}$ by 4; $2\frac{1}{2}$ by 3; $4\frac{1}{4}$ by 2; $1\frac{7}{8}$ by 3; $3\frac{3}{4}$ by 5; $3\frac{3}{5}$ by 6.

148. *Dividing the numerator or multiplying the denominator of a fraction by an integer divides the fraction by that integer.*

Always divide the numerator when it is divisible by the integer.

EXERCISES

149. Divide:

1. $\frac{2}{3}$ by 6	**4.** $\frac{5}{6}$ by 5	**7.** $1\frac{4}{5}$ by 3	**10.** $5\frac{1}{4}$ by 7
2. $\frac{1}{4}$ by 2	**5.** $\frac{3}{8}$ by 6	**8.** $3\frac{1}{3}$ by 2	**11.** $4\frac{1}{6}$ by 5
3. $\frac{3}{5}$ by 3	**6.** $\frac{3}{4}$ by 9	**9.** $7\frac{1}{2}$ by 5	**12.** $3\frac{3}{8}$ by 9

13. If $\frac{7}{8}$ of a bushel of oats will feed a horse 7 times, what part of a bushel will feed him once?

14. If $\$\frac{1}{2}$ is divided equally among 5 boys, what part of a dollar will each boy receive?

15. When 6 dozen eggs cost $\$1\frac{1}{5}$, what is the price per dozen?

16. Eleanor paid $\$2\frac{1}{4}$ for 9 yards of challis. How much did it cost her per yard?

17. Frederick earned $\$5\frac{2}{5}$ in 6 days. How much did he earn per day?

WRITTEN EXERCISES

150. **1.** Divide $\frac{24}{25}$ by 8; $\frac{7}{8}$ by 6; $\frac{15}{16}$ by 10.

$$\frac{24}{25} \div 8 = \frac{24 \div 8}{25} = \frac{3}{25} \qquad \frac{7}{8} \div 6 = \frac{7}{6 \times 8} = \frac{7}{48} \qquad \frac{15}{16} \div 10 = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times 16} = \frac{3}{32}$$

You have learned that dividing the numerator of a fraction divides the fraction; then, since the numerator 24 is exactly divisible by the integer 8, you should divide the numerator by 8. The result is $\frac{3}{25}$.

You have learned that multiplying the denominator of a fraction divides the fraction; then, since the numerator 7 is *not* divisible by the integer 6, you should multiply the denominator by 6. The result is $\frac{7}{48}$.

Usually the *shortest* way to multiply a fraction by an integer is to indicate the multiplication of the denominator by the integer and then cancel whenever possible. Indicating and canceling, the result is $\frac{3}{32}$.

Divide:

2. $\frac{11}{12}$ by 4	**6.** $\frac{3}{4}$ by 24	**10.** $\frac{13}{16}$ by 26
3. $\frac{20}{21}$ by 5	**7.** $\frac{4}{5}$ by 32	**11.** $\frac{5}{18}$ by 25
4. $\frac{24}{35}$ by 8	**8.** $\frac{5}{6}$ by 40	**12.** $\frac{19}{20}$ by 12
5. $\frac{36}{42}$ by 9	**9.** $\frac{7}{8}$ by 42	**13.** $\frac{22}{48}$ by 33

14. Divide $13\frac{3}{4}$ by 5; $213\frac{3}{4}$ by 5.

$13\frac{3}{4} = \frac{55}{4}$

$\frac{55}{4} \div 5 = \frac{11}{4} = 2\frac{3}{4}$

$5)213\frac{3}{4}$

$42\frac{3}{4}$

The integral part of the dividend is *small;* in such a case the mixed number may be reduced to an improper fraction and the division performed as before.

The quotient is $2\frac{3}{4}$.

The integral part of the dividend is *large* and the divisor is small; in this case short division may be used: thus, 5 is contained in $213\frac{3}{4}$, 42 times with $3\frac{3}{4}$, or $\frac{15}{4}$, undivided; $\frac{15}{4} \div 5 = \frac{3}{4}$; then the entire quotient is $42\frac{3}{4}$.

Divide:

15. $18\frac{2}{3}$ by 4

16. $12\frac{5}{6}$ by 7

17. $21\frac{3}{8}$ by 6

18. $53\frac{3}{4}$ by 5

19. $241\frac{1}{2}$ by 3

20. $474\frac{2}{5}$ by 8

21. $738\frac{4}{7}$ by 5

22. $815\frac{5}{8}$ by 9

23. $1347\frac{1}{6}$ by 2

24. $3476\frac{1}{4}$ by 6

25. $5286\frac{7}{8}$ by 9

26. $6304\frac{2}{7}$ by 5

27. Divide $768\frac{3}{4}$ by 15.

$15)\ 768\frac{3}{4}$

$4 \quad 4$

$60)3075(51\frac{1}{4}$

300

75

60

$\frac{15}{60} = \frac{1}{4}$

Here it is necessary to use long division. Before dividing, change both divisor and dividend to fourths obtaining 60 (fourths) and 3075 (fourths), respectively.

Dividing as in integers, the quotient is found to be $51\frac{1}{4}$.

Divide:

28. $246\frac{2}{3}$ by 16

29. $597\frac{3}{5}$ by 24

30. $789\frac{3}{4}$ by 18

31. $408\frac{4}{5}$ by 28

32. $376\frac{5}{6}$ by 21

33. $613\frac{3}{8}$ by 35

34. $54\frac{7}{12}$ by 25

35. $942\frac{6}{7}$ by 75

36. $2619\frac{3}{4}$ by 56

37. $4294\frac{2}{5}$ by 48

38. $9877\frac{1}{8}$ by 63

39. $5054\frac{2}{9}$ by 32

WRITTEN EXERCISES

151. Find in the shortest way:

1. $\frac{7}{12} \div 5$
2. $2\frac{4}{5} \div 8$
3. $9\frac{3}{8} \div 25$
4. $\frac{14}{15} \div 21$
5. $8\frac{2}{5} \div 12$
6. $372\frac{1}{3} \div 6$
7. $654\frac{1}{4} \div 9$
8. $536\frac{5}{8} \div 27$
9. $743\frac{1}{6} \div 35$
10. $942\frac{6}{7} \div 44$
11. $2796\frac{2}{3} \div 4$
12. $4271\frac{3}{4} \div 7$
13. $8429\frac{2}{5} \div 9$
14. $5686\frac{7}{8} \div 45$
15. $829\frac{7}{12} \div 55$

Find the cost of each when

16. 18 chafing dishes cost $\$94\frac{1}{2}$.
17. 36 alarm clocks cost $\$82\frac{4}{5}$.
18. 25 mantel clocks cost $\$78\frac{3}{4}$.
19. 24 cabinet clocks cost $\$87\frac{2}{5}$.
20. 21 opera glasses cost $\$186\frac{9}{10}$.
21. 15 open-face watches cost $\$521\frac{1}{4}$.
22. 33 velour couches cost $\$874\frac{1}{2}$.
23. 52 sewing machines cost $\$998\frac{2}{5}$.
24. 45 cooking stoves cost $\$1091\frac{1}{4}$.
25. 71 nickeled parlor stoves cost $\$1370\frac{3}{10}$.

26. If a train runs $19\frac{1}{5}$ miles in 24 minutes, how far does it run in 1 minute?

27. A bicycle rider went around a track 51 times, thus riding a distance of $12\frac{3}{4}$ miles. How far is it around the track?

28. Mr. Hobbs received $\$1698\frac{3}{4}$ for 45 cows. What was the average price per cow?

29. The receipts of an excursion were $\$3307\frac{1}{2}$. If 945 tickets were sold, what was the fare?

30. If $123\frac{1}{4}$ quarts of milk make 17 pounds of butter, how many quarts of milk are required to make 1 pound of butter?

152. Division of integers by fractions.

1. How many times is $\$\frac{1}{4}$ contained in \$1? in \$2? in \$3? Then $1 \div \frac{1}{4} =$ how many? $2 \div \frac{1}{4} = ?$ $3 \div \frac{1}{4} = ?$

2. What is the value of $1 \div \frac{1}{5}$? $2 \div \frac{1}{5}$? $1 \div \frac{1}{6}$? $3 \div \frac{1}{6}$? $1 \div \frac{1}{3}$? $5 \div \frac{1}{3}$? $4 \div \frac{1}{2}$?

3. Compare the result of $2 \div \frac{1}{3}$ with 2 multiplied by 3; $5 \div \frac{1}{4}$ with 5 multiplied by the denominator of $\frac{1}{4}$. Then how can you divide an integer by a fraction whose numerator is 1?

4. Divide 6 by 1; by 2; by 3. Notice that *as the divisor becomes greater the quotient becomes smaller.*

In $6 \div \frac{1}{5}$ and $6 \div \frac{3}{5}$, which divisor is greater? Which quotient, then, is greater? How can you find the second quotient from the first?

5. What part of $3 \div \frac{1}{4}$ is $3 \div \frac{3}{4}$? From $3 \div \frac{1}{4}$ find $3 \div \frac{3}{4}$.

6. By what number must you divide the result of $2 \div \frac{1}{6}$ to find $2 \div \frac{5}{6}$? Find $2 \div \frac{5}{6}$.

7. Divide 4 by $\frac{2}{3}$; 6 by $\frac{3}{4}$; 3 by $\frac{4}{5}$; 4 by $\frac{5}{6}$.

153. *An integer is divided by a fraction by multiplying the integer by the denominator of the fraction and dividing the product by the numerator.*

EXERCISES

154. Divide: Find:

1. 2 by $\frac{1}{4}$	**4.** 6 by $\frac{2}{3}$	**7.** $5 \div \frac{1}{7}$	**10.** $2 \div \frac{3}{8}$
2. 4 by $\frac{1}{3}$	**5.** 9 by $\frac{3}{4}$	**8.** $4 \div \frac{1}{8}$	**11.** $6 \div \frac{4}{7}$
3. 5 by $\frac{1}{2}$	**6.** 8 by $\frac{4}{5}$	**9.** $7 \div \frac{1}{6}$	**12.** $8 \div \frac{5}{6}$

13. When tea is worth $\$\frac{1}{2}$ a pound, how many pounds can be bought for \$3?

14. How many dozen eggs can be bought for \$2 at $\$\frac{1}{4}$ a dozen?

15. A man pays $\$\frac{3}{4}$ a day for his board. How many days can he board on \$6?

16. If a cobbler can heel a pair of shoes in $\frac{4}{5}$ of an hour, how many pairs can he heel in 8 hours?

17. Mr. Hay divided 5 pounds of candy among his children, giving each $\frac{5}{8}$ of a pound. How many children had he?

18. How many steps will a man take in walking 10 yards, if his step is $\frac{5}{6}$ of a yard long?

155. **1.** Divide 1 by $\frac{3}{4}$ and express the quotient as a fraction.

$1 \div \frac{3}{4} = ?$ $\quad 1 \div \frac{4}{5} = ?$ $\quad 1 \div \frac{2}{3} = ?$ $\quad 1 \div \frac{7}{6} = ?$

2. The fraction $\frac{4}{3}$ is the *reciprocal* of $\frac{3}{4}$; $\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$. What is the reciprocal of $\frac{2}{3}$? of $\frac{7}{6}$?

156. The **reciprocal** of a fraction is 1 divided by the fraction, or it is the fraction inverted.

WRITTEN EXERCISES

157. **1.** Divide 18 by $\frac{4}{5}$.

$$18 \div \frac{4}{5} = \frac{\overset{9}{\cancel{18}} \times 5}{\underset{2}{\cancel{4}}} = \frac{45}{2} = 22\tfrac{1}{2}$$

Since $1 \div \frac{1}{5} = 5$, $18 \div \frac{1}{5} = 18 \times 5$; then, $18 \div \frac{4}{5}$ = $\frac{1}{4}$ of 18×5 (§ 152, 4), or $\frac{18 \times 5}{4}$. Canceling and reducing, the result is found to be $22\frac{1}{2}$.

Observe that dividing 18 by $\frac{4}{5}$ is the same as multiplying 18 by $\frac{5}{4}$. That is, dividing an integer by a fraction is the same as *multiplying the integer by the reciprocal of the fraction.*

Divide:

2. 15 by $\frac{2}{3}$

3. 24 by $\frac{3}{4}$

4. 28 by $\frac{2}{5}$

5. 35 by $\frac{5}{6}$

6. 45 by $\frac{6}{7}$

7. 62 by $\frac{3}{5}$

8. 57 by $\frac{7}{8}$

9. 68 by $\frac{8}{9}$

Find:

10. $78 \div \frac{9}{10}$

11. $81 \div \frac{5}{12}$

12. $95 \div \frac{15}{16}$

13. $63 \div \frac{18}{25}$

14. $245 \div \frac{14}{15}$

15. $456 \div \frac{10}{21}$

16. $372 \div \frac{16}{5}$

17. $657 \div \frac{27}{4}$

18. Divide 28 by $4\frac{3}{8}$.

SUGGESTION. — Reduce the mixed number to an improper fraction and divide as before.

Divide:

19. 21 by $4\frac{1}{2}$	**22.** 48 by $3\frac{3}{5}$	**25.** 126 by $6\frac{3}{4}$
20. 39 by $2\frac{1}{4}$	**23.** 65 by $6\frac{1}{6}$	**26.** 395 by $5\frac{5}{6}$
21. 44 by $6\frac{2}{3}$	**24.** 90 by $1\frac{7}{8}$	**27.** 625 by $9\frac{3}{8}$

28. Divide 225 by $18\frac{3}{4}$.

$18\frac{3}{4})225$
$\quad 4 \quad\; 4$
$75\ )900(12$

By changing both dividend and divisor to *fourths*, the numerator 900 may be divided by the numerator 75 as in integers.

In some cases, especially when the divisor is large, it is convenient to employ this method.

Divide:

29. $345 \div 27\frac{3}{5}$	**31.** $725 \div 45\frac{5}{6}$	**33.** $609 \div 14\frac{7}{10}$
30. $528 \div 38\frac{2}{5}$	**32.** $910 \div 56\frac{7}{8}$	**34.** $825 \div 27\frac{1}{12}$

WRITTEN EXERCISES

158. Find by the method best adapted:

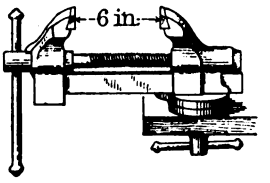
1. $17 \div \frac{1}{4}$	**5.** $164 \div \frac{6}{7}$	**9.** $775 \div 20\frac{5}{6}$
2. $31 \div \frac{2}{3}$	**6.** $327 \div \frac{7}{8}$	**10.** $925 \div 34\frac{3}{8}$
3. $64 \div \frac{5}{12}$	**7.** $462 \div \frac{15}{16}$	**11.** $2396 \div 6\frac{2}{5}$
4. $97 \div 8\frac{1}{6}$	**8.** $695 \div 3\frac{3}{4}$	**12.** $5285 \div 46\frac{2}{3}$

13. There are $16\frac{1}{2}$ feet in a rod. How many rods are there in 792 feet?

14. If a glacier in the Alps moves at an average rate of $1\frac{2}{3}$ feet a day, how many days will it take to move 660 feet, or an eighth of a mile?

15. How many pounds of cotton must a man pick to earn \$1, if he receives $\$28\frac{4}{5}$ for picking 7200 pounds?

16. If each turn of the screw brings the jaws of this vise $\frac{3}{16}$ in. nearer together, how many turns will it take to close the vise?



17. If it requires $4\frac{3}{4}$ yards of silk to make a waist, how many waists can be made from a piece containing 38 yards?

18. A barrel of flour weighs 196 pounds. If it takes $\frac{3}{4}$ of a pound for a loaf of bread, how many loaves can a baker make out of 3 barrels of flour?

19. A man and his wife took a trip in a gasoline launch, sailing 6 hours a day. The launch went at the rate of $8\frac{5}{8}$ miles an hour, and covered a distance of 1311 miles. How many days were they on the trip?

159. Finding the whole when a fractional part of it is given.

EXERCISES

1. If $\frac{3}{5}$ of the cost of a ball is 30 cents, find the whole cost.

SOLUTION. $\frac{3}{5}$ of the cost $= 30¢$.
$\frac{1}{5}$ of the cost $= \frac{1}{3}$ of $30¢$, or $10¢$.
$\frac{5}{5}$ of the cost $= 5 \times 10¢$, or $50¢$.

2. Helen paid 12 cents for $\frac{3}{4}$ of a pound of mixed nuts. At that rate how much would a pound cost?

3. Our baseball team won $\frac{2}{3}$ of the games that it played. If it won 14 games, how many did it play?

4. How deep is a well that goes through 18 feet of rock, if $\frac{3}{8}$ of its depth is through rock?

5. If $\frac{5}{6}$ of the passengers on a car were men, and there were 35 men, how many passengers were there?

6. A football player ran $\frac{2}{5}$ of the length of the field. If he ran 66 yards, how long was the field?

WRITTEN EXERCISES

160. **1.** If 679 is $\frac{7}{8}$ of a number, what is the number?

SOLUTION. $\frac{7}{8}$ of the number = 679.

$\frac{1}{8}$ of the number = $\frac{1}{7}$ of 679, or 97.

$\frac{8}{8}$ of the number = 8×97, or 776.

Or the work may be done more quickly by indicating it and then canceling; thus:

$$679 \div \frac{7}{8} = \frac{\overset{97}{\cancel{679}} \times 8}{\cancel{7}} = 776.$$

Find the number of which

2. 68 is $\frac{2}{3}$	**5.** 224 is $\frac{7}{10}$	**8.** 1347 is $\frac{3}{8}$
3. 81 is $\frac{3}{5}$	**6.** 605 is $1\frac{1}{12}$	**9.** 4684 is $\frac{4}{5}$
4. 95 is $\frac{5}{6}$	**7.** 747 is $\frac{9}{16}$	**10.** 8298 is $\frac{6}{7}$

11. How far is it from Chicago to Detroit, if $\frac{3}{5}$ of the distance is 171 miles?

12. A man's expenses are $\frac{4}{5}$ of his earnings. If his expenses are \$816 a year, how much does he earn?

13. If a steer when killed and dressed weighs $\frac{7}{10}$ as much as when alive, what was the live weight of one that dressed 742 pounds?

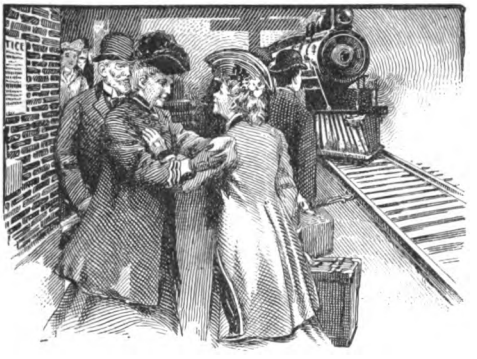
14. If a duck, flying $\frac{3}{5}$ as fast as a hawk, flies 90 miles in an hour, how fast does the hawk fly?

15. A printing press can print $\frac{3}{8}$ as many copies of a 20-page paper as of an 8-page paper. If it prints 36,000 20-page papers per hour, how many 8-page papers will it print in that time?

16. The bell of the Houses of Parliament in London weighs 30,000 pounds, and is $\frac{5}{26}$ as heavy as the heaviest bell in the world, which is at Moscow. How much does the bell at Moscow weigh?

PROBLEMS IN REVIEW

161. **1.** When purchasing Ada's boarding-school outfit, her mother examines two trunks, one 36 in. by $20\frac{1}{2}$ in. by 23 in., the other 34 in. by 21 in. by 24 in. How many more cubic inches does the larger trunk contain than the smaller one?



2. The first trunk weighs 63 lb., the second 56 lb. Compare their weights in two ways.

3. At one store she selects 2 suits at \$10.75 each, a walking skirt for \$6.50, 3 shirtwaists at \$2.25 each, and a Sunday suit for \$35. Find her change from four 20-dollar bills.

4. At another store she buys a dozen towels and a dozen wash cloths for \$3.30. If the price of the wash cloths is 2 for 5¢, what is the price of each towel?

5. She orders 2 pairs of shoes at \$3.50 a pair and a pair of Oxford ties for \$2.50. Find the cost of shoes and ties.

6. At Hill's she buys a pair of kid gloves for \$1.25, a dozen pairs of stockings at 3 for \$1, a dozen handkerchiefs at $12\frac{1}{2}$¢ each, 2 stocks at 49¢ each, 2 belts at 35¢ each, and $6\frac{3}{4}$ yards of ribbon at 16¢ a yard. Find the cost.

7. Find the amount of these additional purchases:

Sunday hat,	\$6.00	Raincoat,	\$12.00	Party dress,	\$18.00
Week-day hat,	2.50	Umbrella,	3.00	Silk waist,	6.00
Toilet articles,	5.19	Overshoes,	.65	Miscellaneous,	30.67

8. Find the total cost of fitting out Ada for boarding school, including \$6.75 for a trunk and \$2.49 for a suit case.

9. The Girls' Sunshine Club held a fair for a Fresh Air Fund. Elsa made for it 10 lb. stuffed dates, paying 15¢ a pound for the dates, 72¢ a peck for the peanuts, and 5¢ for the sugar. If 2 lb. dates and 2 qt. peanuts make $2\frac{1}{2}$ lb. stuffed dates, what quantities did Elsa buy and how much did her contribution cost her?

10. Miriam made 5 lb. cream walnuts. She used the whites of 4 eggs, as much water as egg, 2 lb. sugar, 3 lb. English walnuts, and 5 cents' worth of vanilla flavoring. Find the cost if eggs were 24¢ a dozen, confectioners' sugar 8¢ a pound, and walnuts 18¢ a pound.

11. Helen made 12 fern baskets, paying 36¢ for the reed and 10¢ apiece for the ferns to put into the baskets. What was the cost of each filled basket?

12. Alice made 5 laundry bags, using $1\frac{1}{4}$ yd. of toweling and 3 yd. of ribbon for each bag. At 40¢ a yard for the toweling, and 10¢ a yard for the ribbon, find the cost of all the bags.

13. The stuffed dates were sold at 40¢ a pound, the cream walnuts at 45¢ a pound, the fern baskets at 20¢ apiece, and the laundry bags at $1.25 apiece. The receipts from the refreshment booth were $7.65, from the flower booth $9.35, and from other booths $37.15. Find the total receipts from the fair.

DECIMAL FRACTIONS

162. Review definitions.

1. Fractions that express tenths or hundredths or thousandths, etc., are called either **decimal fractions**, or simply **decimals.**

The term "decimals" is more commonly applied to decimal fractions that are expressed with a *decimal point.*

.3 and .25 are decimal fractions, or decimals.

2. Fractions that do not express tenths, or hundredths, or thousandths, etc., are called **common fractions.**

$\frac{1}{8}$, $\frac{25}{32}$, $\frac{9}{16}$ are common fractions.

Reduction of Decimals

163. Reduction of decimals to common fractions.

WRITTEN EXERCISES

1. Reduce 7.025 to a mixed number having its decimal part expressed as a common fraction in its lowest terms.

$$7.025 = 7\frac{25}{1000} = 7\frac{1}{40}.$$

Reduce the following to common fractions in their lowest terms, or to mixed numbers with the fractional part in lowest terms:

2. .5	7. .15	12. 7.05	17. 7.625
3. .4	8. .25	13. 3.75	18. 4.875
4. 1.6	9. .32	14. .125	19. 9.324
5. 7.8	10. .65	15. .375	20. 6.075
6. 8.4	11. .95	16. .875	21. 5.012

164. Reduction of common fractions to decimals.

1. How many tenths are there in 1? in 3? Then how many tenths are there in $\frac{1}{5}$ of 3, or in $\frac{3}{5}$?

$5\overline{)30}$ tenths
6 tenths

$5\overline{)3.0}$
.6

2. How many hundredths are there in 3? Then how many hundredths are there in $\frac{1}{4}$ of 3, or in $\frac{3}{4}$?

$4\overline{)300}$ hundredths
** hundredths

$4\overline{)3.00}$
.**

3. Reduce $\frac{5}{8}$ to thousandths in this way:

$8\overline{)5000}$ thousandths
*** thousandths

$8\overline{)5.000}$
.***

WRITTEN EXERCISES

165. 1. Reduce $\frac{2}{3}$ to a decimal.

$3\overline{)2.000}$
$.666\frac{2}{3}$

Since $\frac{2}{3} = 2 \div 3$, divide 2 by 3.
$\frac{1}{3}$ of 20 tenths = 6 tenths and 2 tenths remaining.
2 tenths = 20 hundredths.
$\frac{1}{3}$ of 20 hundredths = 6 hundredths and 2 hundredths remaining.
2 hundredths = 20 thousandths.
$\frac{1}{3}$ of 20 thousandths = $6\frac{2}{3}$ thousandths.
Therefore, $\frac{2}{3} = .666\frac{2}{3}$.

One decimal cipher is annexed for each decimal place required. At present results need not be carried beyond three decimal places.

Reduce to decimals:

2. $\frac{2}{5}$	7. $\frac{3}{8}$	12. $\frac{5}{6}$	17. $\frac{5}{12}$
3. $\frac{3}{5}$	8. $\frac{5}{8}$	13. $\frac{2}{7}$	18. $\frac{17}{20}$
4. $\frac{4}{5}$	9. $\frac{7}{8}$	14. $\frac{4}{9}$	19. $\frac{13}{500}$
5. $\frac{1}{4}$	10. $\frac{1}{3}$	15. $\frac{7}{9}$	20. $\frac{77}{200}$
6. $\frac{1}{2}$	11. $\frac{1}{6}$	16. $\frac{3}{11}$	21. $\frac{6}{700}$

Addition and Subtraction of Decimals

WRITTEN EXERCISES

166. The following have been added and tested in 18 minutes. Practice until you can do as well.

1.	2.	3.	4.
14.386	27.909	642.8	6.775
7.545	6.7	37.95	29.22
12.564	85.387	42.44	63.72
90.387	146.962	68.308	698.285

5.	6.	7.	8.
49.837	45.555	72.499	71.473
37.922	77.6	36.309	66.206
256.39	60.72	108.758	48.68
70.683	84.966	728.025	92.375
96.007	67.386	88.989	190.806

9.	10.	11.	12.
$868.369	$ 89.73	$ 77.49	$4708.95
397.487	448.29	345.18	16.884
549.2	569.375	806.725	472.492
305.868	92.447	49.875	5848.13
844.796	999.96	72.125	429.418
84.34	47.775	450.50	484.38
256.634	38.248	81.475	91.656

13.	14.	15.	16.
913.216	$ 75.45	$194.19	$9999.99
300.999	888.88	63.72	8888.88
20.075	8.09	.05	7777.77
.009	459.68	80.09	.66
154.073	43.35	908.05	5.55
13.95	799.99	5.17	44.44
621.79	81.04	64.31	3333.33

Write, with units of the same order in the same column, add, and test results. Preserve your answers.

17. $3.656 + 4.381 + 49.244 + 58.9 + 76.75 + 97.975$

18. $6.258 + 3.849 + 66.495 + 37.4 + 49.25 + 9.346$

19. $7.437 + 9.000 + 42.835 + 56.7 + 38.28 + 45.305$

20. $4.888 + 7.495 + 13.625 + 98.5 + 49.56 + 99.999$

21. Add the sums obtained in exercises 17–20.

22. Add each of the six columns seen in exercises 17–20; then add the six sums thus obtained.

The answer should be the same as for exercise 21.

23. Add: seventy and six tenths; forty-five and nine hundredths; six hundred and sixteen hundredths; six hundred sixteen; six hundred sixteen thousandths; five thousandths.

24. Add: four hundred fifty-six; four hundred and fifty-six thousandths; nine thousand and ninety-five hundredths; eighty-six and eighty-six thousandths.

Subtract:

25. 728.249 575.655	**26.** 400.289 38.491	**27.** 540.00 235.68	**28.** 408.30 182.94
29. 200.00 96.48	**30.** 700 47.75	**31.** 412 62.628	**32.** 5247 399.99
33. 428 73.86	**34.** 525.4 96.275	**35.** 600.02 48.003	**36.** 500 28.265
37. 1000 39.25	**38.** 1000 56.625	**39.** 1000 107.112	**40.** 1000 11.089

WRITTEN EXERCISES

167. **1.** How much larger is the English bushel of 2218.192 cu. in. than the American bushel of 2150.42 cu. in.?

2. A gallon of water weighs 8.355 pounds, and a gallon of milk 8.622 pounds. Which is the heavier, and how much?

3. A chimney stands 220.25 ft. above ground and extends 18.50 ft. below, where it rests on a foundation 7.75 ft. deep. Find the total height of chimney and foundation.

4. A baseball was thrown from the home plate to second base, 127.28 ft.; then from second base to first base, 90 ft.; then from first base to third base, 127.28 ft. How far did the ball travel?

5. The Times building extends 143 ft. along Broadway, 58.33⅓ ft. along 42d Street, 137.83⅓ ft. along 7th Avenue, and 20 ft. along 43d Street. Find the distance around it.

6. The increase in price of cotton during one season lasted eight months. The average price per pound each month was:

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.
8.95¢	9.65¢	10.08¢	10.44¢	11.46¢	12.40¢	12.74¢	12.75¢

Find the advance in price from each month to the next.

Test your results by adding them and comparing the sum with the total advance of the August price over the January price.

7. One year the rainfall in a certain section of Oregon was 35.69 in. during the first four months, 9.47 in. the next four, and 31.82 the last four. Find the rainfall for the year.

8. A certain railroad had 1631.03 miles of track in Illinois, 1472.99 in Iowa, 1388.58 in Missouri, 233.10 in Wisconsin, 38.45 in Minnesota, 2632.25 in Nebraska, 199.90 in Montana, 260.44 in Kansas, 429.35 in Colorado, 286.66 in South Dakota, and 351.53 in Wyoming. Find the total mileage.

Multiplication of Decimals

WRITTEN EXERCISES

168. Multiply:

1. $256.75 × 4 = $1027.00	**2.** 256.75 × 12 = 3081.00	**3.** 437.45 × 6 = 2624.70	**4.** 95.364 × 5 = 476.820

Ciphers at the right of a decimal may be *omitted*.

5. $614.38 × 5	**6.** 427.65 × 8	**7.** $396.25 × 12	**8.** 66.666 × 5
9. 306.46 × 11	**10.** 38.695 × 9	**11.** 40.625 × 8	**12.** 79.694 × 7

13. Multiply 42.46 by $6\frac{2}{3}$.

$$\begin{array}{r} 42.46 \\ 6\frac{2}{3} \\ \hline 3)\overline{84.92} \\ 28.30\frac{2}{3} \\ 254.76 \\ \hline 283.06\frac{2}{3} \end{array}$$

42.46 multiplied by $6\frac{2}{3}$ is equal to the sum of $\frac{2}{3}$ of 42.46 and 6 times 42.46.

$\frac{2}{3}$ of 42.46 = $\frac{1}{3}$ of 2 times 42.46
= $\frac{1}{3}$ of 84.92
= $28.30\frac{2}{3}$
$6 \times 42.46 = 254.76$

Therefore the product is $28.30\frac{2}{3} + 254.76$, or $283.06\frac{2}{3}$.

Multiply:

14. 37.72 by $8\frac{2}{5}$	**20.** 385.61 by $7\frac{5}{6}$
15. 4.875 by $6\frac{3}{4}$	**21.** 94.395 by $11\frac{7}{8}$
16. 9.624 by $9\frac{5}{8}$	**22.** 87.366 by $12\frac{4}{5}$
17. 83.55 by $7\frac{1}{5}$	**23.** 342.99 by $10\frac{1}{9}$
18. 7.283 by $6\frac{2}{3}$	**24.** 81.128 by $12\frac{3}{8}$
19. 6.364 by $5\frac{2}{7}$	**25.** 71.006 by $11\frac{3}{5}$

169. 1. How many thousandths make 1 hundredth? How many hundredths make 1 tenth? How many tenths make 1 unit?

2. Since 10 times .1 = 1, what is the value of

$10 \times .2$? $10 \times .3$? $10 \times .4$? $10 \times .5$?

Multiplying tenths by 10 *gives units.*

3. Since 10 times .01 = .1, what is the value of

$10 \times .03$? $10 \times .05$? $10 \times .07$? $10 \times .08$?

Multiplying hundredths by 10 *gives tenths.*

4. Since 10 times .001 = .01, what is the value of

$10 \times .005$? $10 \times .006$? $10 \times .012$? $10 \times .015$?

Multiplying thousandths by 10 *gives hundredths.*

5. Since $10 \times .005 = .05$, $10 \times .03 = .3$, and $10 \times .2 = 2$, when .235 is multiplied by 10, each figure of .235 is advanced into the next higher place. A short way to do this is to *move the decimal point one place toward the right.*

$$\begin{array}{r} .235 \\ 10 \\ \hline 2.350 \end{array}$$

6. Multiply .235 by 10 and the product by 10, and so obtain $100 \times .235$. Multiply by 10 again and obtain $1000 \times .235$.

How many places and in which direction is the decimal point moved for each multiplication by 10? How may a decimal be multiplied by 100 by moving the decimal point? by 1000?

170. *A decimal may be multiplied by* 10, 100, 1000, *etc., by moving the decimal point toward the right as many places as there are ciphers in the multiplier.*

EXERCISES

Multiply by 10:

1. .4
2. .35
3. .09
4. $4.65
5. $3.96
6. $0.075

Multiply by 100:

7. .48
8. 3.90
9. 0.009
10. 5.315
11. 7.56
12. 3.004

Multiply by 1000:

13. .016	15. 9.060	17. $5.75	19. $2.375
14. 3.050	16. 4.06	18. $2.16	20. 230.6

Multiply by 10, 100, 1000:

21. 3.025	23. 7.011	25. 15.603	27. 100.11
22. 4.365	24. 8.009	26. 243.62	28. 4256.3

29. Multiply .785 by 1000; by 10,000; by 100,000.

WRITTEN EXERCISES

171. 1. Multiply 7.628 by 200.

$$\begin{array}{r} 762.8 \\ 2 \\ \hline 1525.6 \end{array}$$

100 times 7.628 = 762.8.
200 times 7.628 = 2 times 762.8.
= 1525.6.

Multiply by 20, 30, 40, 50, 60:

2. 3.42	4. 4.48	6. $2.365	8. 87.1
3. .567	5. .007	7. $24.82	9. 48.6

Multiply by 700, 800, 900, 1200:

10. .382	12. $4.29	14. 25.05	16. 79.6
11. .796	13. $16.41	15. 122.8	17. 40.9

Multiply by 6000, 70,000, 11,000,000:

18. 2.254	19. .079	20. $4.18	21. 9.9

22. Multiply $48.57 by 84; 3.845 by 96.

$$\begin{array}{r} \$48.57 \\ 84 \\ \hline 194\ 28 \\ 3885\ 6 \\ \hline \$4079.88 \end{array} \qquad \begin{array}{r} 3.845 \\ 96 \\ \hline 23\ 070 \\ 346\ 05 \\ \hline 369.12\emptyset \end{array}$$

Dollar signs and decimal points are omitted in the partial products.

Multiply:

23. \$36.62 by 89
24. \$94.05 by 66
25. 37.088 by 75
26. 42.094 by 69
27. 78.889 by 98
28. 39.675 by 125
29. 84.719 by 280
30. 78.616 by 675
31. 90.008 by 845
32. 468.77 by 1208

33. Find the weight of a slate roof 40 feet long and 20 feet wide, if every square foot weighs 9.25 pounds.

34. If it takes 7.5 cu. ft. of natural gas to produce as much heat as 1 pound of coal, how many cubic feet of gas are required to give as much heat as 1 ton of coal?

35. A cubic foot of water weighs 62.5 lb. Find the weight of water that a tank 8 ft. by 4 ft. by 3 ft. will hold.

Division of Decimals

WRITTEN EXERCISES

172. 1. $7)\overline{378.240}$ $= 54.034\frac{2}{7}$

2. $9)\overline{456.392}$ $= 50.710\frac{2}{9}$

3. $12)\overline{62.574}$ $= 5.214\frac{1}{2}$

Divide, finding quotients to three decimal places:

4. 34.234 by 6
5. 56.712 by 8
6. 95.203 by 7
7. 84.425 by 9
8. 50.806 by 7
9. 61.753 by 9
10. 5.8 (or 5.800) by 8
11. 7.6 by 11
12. 345.5 by 9
13. 46.37 by 7
14. 1.044 by 12
15. 3.058 by 11

16. Find $\frac{1}{8}$ of 4.512; of 6.2; of \$14.64.

17. Find $\frac{1}{12}$ of \$17.28; of 72.096; of 11.20.

173. **1.** How many tenths are there in $\frac{1}{10}$ of 10 tenths, or in $\frac{1}{10}$ of 1? in $\frac{1}{10}$ of 2? in $\frac{1}{10}$ of 3?

$1 \div 10 = .1$ $2 \div 10 = ?$ $3 \div 10 = ?$

Dividing units by 10 *gives tenths.*

2. How many hundredths are there in $\frac{1}{10}$ of .10, or in $\frac{1}{10}$ of .1? in $\frac{1}{10}$ of .4? in $\frac{1}{10}$ of .5?

$.1 \div 10 = .01$ $.4 \div 10 = ?$ $.5 \div 10 = ?$

Dividing tenths by 10 *gives hundredths.*

3. How many thousandths are there in $\frac{1}{10}$ of .010, or in $\frac{1}{10}$ of .01? in $\frac{1}{10}$ of .06? in $\frac{1}{10}$ of .07?

$.01 \div 10 = .001$ $.08 \div 10 = ?$ $.09 \div 10 = ?$

Dividing hundredths by 10 *gives thousandths.*

4. Since $2 \div 10$, or $\frac{1}{10}$ of 2, is equal to .2, $\frac{1}{10}$ of $.5 = .05$, and $\frac{1}{10}$ of $.06 = .006$, dividing 2.56 by 10 has the effect of moving each figure into the next lower place. A short way to do this is to *move the decimal point one place toward the left.*

$10\overline{)2.56}$
$.256$

5. If 2.5 is divided by 10 and the quotient by 10, how many places toward the left will the decimal point be moved?

How may a number be divided by 10×10, or by 100?

6. Divide 3, or 3.0, by 10; divide the quotient by 10; divide a third time by 10. How many places and in which direction has the decimal point been moved?

How may a number be divided by 1000?

174. *A number may be divided by* 10, 100, 1000, *etc., by moving the decimal point toward the left as many places as there are ciphers in the divisor.*

EXERCISES

175. Divide:

By 10	By 100	By 1000
1. 72	8. 225	15. 3244
2. 3.5	9. 47.6	16. 8640
3. 4.2	10. 39.6	17. 966
4. .8	11. 1.07	18. 170
5. .09	12. 58.2	19. 49
6. .75	13. 396	20. 5
7. 4.25	14. 44.05	21. 6220

WRITTEN EXERCISES

176. 1. Divide 568 by 30.

$30)56|8.00$
$18.93\frac{1}{3}$

30 is contained in 568 as many times as 3 is contained in 56.8.

Dividing 56.8 by 3, the division is found to be inexact in tenths' place. Therefore, one cipher is annexed to the dividend, and the division is carried to hundredths. The division being inexact in hundredths' place, another cipher is annexed and the division is carried to thousandths, giving 3 thousandths and 1 thousandth remainder.

It is not necessary to carry this division beyond thousandths. The result of the last division is written in the quotient as $3\frac{1}{3}$ thousandths, or the quotient to thousandths may be written 18.933.

In the following exercises, find quotients to thousandths.

Divide by 20, 40, 60, 80, 120:

2. 4	4. .5	6. $35	8. .96
3. 71	5. 4.2	7. $5.60	9. 3.63

Divide by 700, 900, 1100:

10. 25	12. $2763	14. 37.5	16. .6
11. 160	13. $4840	15. 51.2	17. 14.4

Divide by 8000, 11,000, 12,000:

18. 42	19. 132	20. 1680	21. 1056

22. Divide 9.588 by 47; \$10,010 by 125.

$$\begin{array}{r} .204 \\ 47\overline{)9.588} \\ 94 \\ \hline 188 \\ 188 \\ \hline \end{array} \qquad \begin{array}{r} 80.08 \\ 125\overline{)\$10010.00} \\ 1000 \\ \hline 1000 \\ 1000 \\ \hline \end{array}$$

Divide:

23. 292.4 by 85

24. 811.22 by 98

25. \$400 by 625

26. \$328.80 by 48

27. \$860.30 by 176

28. 300.933 by 87

29. 141.401 by 327

30. 351.14 by 388

31. 16637.5 by 275

32. 1449.02 by 265

Reduce to decimals, not beyond thousandths:

33. $\frac{5}{16}$ **34.** $\frac{11}{24}$ **35.** $\frac{3}{32}$ **36.** $\frac{153}{48}$ **37.** $\frac{100}{64}$

38. A man sheared 88 Angora goats and obtained 330 pounds of mohair. Find the average clip from a goat.

39. A 35-acre vineyard produced 259 tons of grapes, which were sold for \$4014.50. Find the average yield per acre, and the value of the crop per ton and per acre.

40. A watch factory employed 2400 persons 8 hours a day. The weekly pay-roll was \$20,160. Find the average wage per hour in cents.

41. During April Mr. Hanna's 30 cows produced 18,800 lb. of milk. Find the average number of pounds of milk produced by 1 cow in 1 day.

42. The milk contained 656 pounds of butter fat, which was sold at 27¢ a pound. Find the average value of the butter fat produced by 1 cow in 1 day.

USEFUL PARTS OF A DOLLAR

177. Halves, fourths, and eighths of a dollar.

1. What part of a dollar is \$.50? \$.25? \$.75?

2. Since \$.25 is $\frac{1}{4}$ of a dollar, what part of a dollar is $\frac{1}{2}$ of \$.25, or \$.12$\frac{1}{2}$?

3.

\$.12$\frac{1}{2}$	\$.25	\$.37$\frac{1}{2}$	\$.50	\$.62$\frac{1}{2}$	\$.75	\$.87$\frac{1}{2}$
+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$	+.12$\frac{1}{2}$
\$.25	\$.37$\frac{1}{2}$	\$.50	\$.62$\frac{1}{2}$	\$.75	\$.87$\frac{1}{2}$	\$1.00

How many eighths of a dollar is \$.12$\frac{1}{2}$ + \$.12$\frac{1}{2}$, or \$.25? \$.25 + \$.12$\frac{1}{2}$, or \$.37$\frac{1}{2}$? \$.50? \$.62$\frac{1}{2}$? \$.75? \$.87$\frac{1}{2}$?

4. Count by eighths of a dollar to \$1.25 in this way:

\$.12$\frac{1}{2}$ = \$$\frac{1}{8}$; \$.25 = \$$\frac{2}{8}$, or \$$\frac{1}{4}$; \$.37$\frac{1}{2}$ = \$$\frac{3}{8}$; etc.

EXERCISES

178. 1. Find the cost of 32 hats @ \$.37$\frac{1}{2}$.

SOLUTION. — At \$1 each, 32 hats would cost \$32.
Therefore at \$.37$\frac{1}{2}$, or \$$\frac{3}{8}$, each, 32 hats will cost $\frac{3}{8}$ of \$32, or \$12.

Find the cost of:

2. 28 sleds @ \$.50.
3. 54 knives @ \$.50.
4. 8 baseball bats @ \$.25.
5. 14 shin guards @ \$.25.
6. 24 neckties @ \$.75.
7. 40 mufflers @ \$.75.
8. 48 yd. lawn @ \$.12$\frac{1}{2}$.
9. 84 yd. gingham @ \$.12$\frac{1}{2}$.
10. 48 tennis balls at \$.37$\frac{1}{2}$.
11. 32 pr. tennis shoes @ \$.37$\frac{1}{2}$.
12. 30 pocketbooks @ \$.62$\frac{1}{2}$.
13. 400 bass lines @ \$.62$\frac{1}{2}$.
14. 80 hammocks @ \$.87$\frac{1}{2}$.
15. 64 boy's sweaters @ \$.87$\frac{1}{2}$.

16. Write 24 in the empty square. Then tell the cost of 24 articles at the prices named.

17. Do the same with the following numbers written successively in the empty square: 32, 48, 72, 80, 96.

\$1	\$.12½	\$.25
\$.87½		\$.37½
\$.75	\$.62½	\$.50

179. Thirds and sixths of a dollar.

1. What part of a dollar is \$.33⅓? ½ of \$.33⅓, or \$.16⅔?

2. Since \$.33⅓ is ⅓ of a dollar, what part of a dollar is \$.33⅓ + \$.33⅓, or \$.66⅔?

3	\$1.00
2	\$ .33⅓
	\$ .16⅔

3.

\$.16⅔	\$.33⅓	\$.50	\$.66⅔	\$.83⅓
+ .16⅔	+ .16⅔	+ .16⅔	+ .16⅔	+ .16⅔
\$.33⅓	\$.50	\$.66⅔	\$.83⅓	\$1.00

How many sixths of a dollar is \$.33⅓? \$.50? \$.66⅔? \$.83⅓?

4. Count by sixths of a dollar to \$1.33⅓ in this way:

\$.16⅔ = \$⅙; \$.33⅓ = \$⅓; \$.50 = \$½, etc.

EXERCISES

180. Find the value of each of the following lots of cloth:

1. 60 yd. @ \$.33⅓.
2. 42 yd. @ \$.16⅔.
3. 27 yd. @ \$.66⅔.
4. 36 yd. @ \$.16⅔.
5. 120 yd. @ \$.33⅓.
6. 600 yd. @ \$.83⅓.
7. 360 yd. @ \$.66⅔.
8. 144 yd. @ \$.83⅓.
9. Find the cost of 1200 baskets @ \$.16⅔.
10. Find the cost of 27 doz. crocks @ \$.66⅔.
11. Find the cost of 24 gross of clothespins @ \$.33⅓.

181. Commit to memory the following table showing the decimal parts of $1, or of any other unit, that are advantageously changed to the common fractional form in computations.

TABLE

PARTS OF $1	PARTS OF $1	PARTS OF $1	PARTS OF $1
$.10 = \frac{1}{10}$	$.80 = \frac{4}{5}$	$.50 = \frac{1}{2}$	$.16\frac{2}{3} = \frac{1}{6}$
$.20 = \frac{1}{5}$	$.12\frac{1}{2} = \frac{1}{8}$	$.62\frac{1}{2} = \frac{5}{8}$	$.33\frac{1}{3} = \frac{1}{3}$
$.40 = \frac{2}{5}$	$.25 = \frac{1}{4}$	$.75 = \frac{3}{4}$	$.66\frac{2}{3} = \frac{2}{3}$
$.60 = \frac{3}{5}$	$.37\frac{1}{2} = \frac{3}{8}$	$.87\frac{1}{2} = \frac{7}{8}$	$.83\frac{1}{3} = \frac{5}{6}$

WRITTEN EXERCISES

182. Find the cost of:

1. 60 plates @ $.10
2. 34 yd. braid @ $.20
3. 36 collars @ $.12½
4. 64 lb. pecans @ $.12½
5. 140 yd. percale @ $.12½
6. 135 baseball bats @ $.20
7. 110 baseball mitts @ $.50
8. 144 bottles olives @ 25¢
9. 66 thimbles @ 33⅓¢
10. 18 handkerchiefs @ 25¢
11. 22 hair brushes @ 37½¢
12. 42 toy engines @ 62½¢
13. 75 yards of sateen @ 16⅔¢
14. 54 yards of damask @ 66⅔¢
15. 140 gallons of sirup @ 37½¢
16. 125 phonograph records @ $.50
17. Find the cost of 15 whips at $1.25 each.

$15
3.75
$18.75

At $1 each, 15 whips would cost $15.
At $1.25, or $1¼, each, they will cost ¼ of $15, or $18.75.
The business man, however, obtains the answer in a more simple way. He first writes $15, the cost at $1 each; then below $15 he writes $3.75, the cost at $.25, or $¼, found by dividing $15 by 4. Adding the cost at $.25 to the cost at $1, he obtains the cost at $1.25.

18. Find the cost of 44 yards of silk at $\$1.12\frac{1}{2}$ per yard.

$\$44$
5.50
$\$49.50$

What is the cost at \$1 per yard? at $\$.12\frac{1}{2}$, or $\$\frac{1}{8}$, per yard? at $\$1.12\frac{1}{2}$ per yard?

19. Find the cost of 22 yards of carpet at $\$.87\frac{1}{2}$ per yard.

$\$22$
2.75
$\$19.25$

What is the cost at \$1 per yard? at $\$.12\frac{1}{2}$, or $\$\frac{1}{8}$, per yard? at $\$1 - \$.12\frac{1}{2}$, or $\$.87\frac{1}{2}$, per yard?

20. How much must be paid for 3548 bushels of wheat at $90\,¢$ per bushel?

$\$3548$
354.80
$\$3193.20$

How much would the wheat cost at \$1 per bushel? at $10\,¢$, or $\$\frac{1}{10}$, per bushel? What, then, is the cost at $90\,¢$, or at \$1 − \$.10, per bushel?

Find the cost of:

21. 56 croquet sets @ \$1.50

22. 36 tennis nets @ \$1.25

23. 45 tennis rackets @ \$2.50

24. 15 baseballs @ \$1.25

25. 24 bathing suits @ \$1.75

26. 33 fishing rods @ \$2.25

27. 420 caps @ $37\frac{1}{2}\,¢$

28. 58 yd. silk @ $\$1.12\frac{1}{2}$

29. 63 yd. serge @ $83\frac{1}{3}\,¢$

30. 48 yd. velvet @ $\$1.16\frac{2}{3}$

31. 28 lawn settees @ $87\frac{1}{2}\,¢$

32. 120 toy watches @ \$.40

33. Find the cost of 45 articles @ $20\,¢$; @ $60\,¢$; @ $80\,¢$.

34. Find the cost of 288 articles @ $50\,¢$; @ $51\,¢$; @ $49\,¢$.

35. Find the cost of 216 articles @ $25\,¢$; @ $26\,¢$; @ $24\,¢$; @ $25\frac{1}{2}\,¢$; @ $24\frac{1}{2}\,¢$.

36. Find the cost of 240 articles @ $33\frac{1}{3}\,¢$; @ $33\,¢$; @ $33\frac{1}{2}\,¢$.

37. A grocer bought 650 pounds of coffee at $22\frac{1}{2}$ cents per pound, and sold it at 35 cents per pound. How much did he gain?

EXERCISES

183. 1. How many articles costing 50¢ each can be bought for each of the following sums:

$1? $2? $5? $12? $1.50? $3.50? $7.50?

2. How many 25-cent articles can be bought for:

$1? $4? $.50? $2 + $.25? $3.50?
$2? $11? $.75? $2 + $.75? $6.25?

3. How many $12\frac{1}{2}$-cent articles can be bought for:

$1? $5? $.25? $1 + $.50? $3.25?
$3? $12? $2 + $.25? $2 + $.75? $5.75?

4. How many $33\frac{1}{3}$-cent articles can be bought for:

$1? $4? $10? $13? $33?
$2? $6? $12? $15? $50?

How many $16\frac{2}{3}$-cent articles can be bought for the same amounts?

WRITTEN EXERCISES

184. 1. How many yards of silk costing $1.12\frac{1}{2}$ per yard can be bought for $315?

$$\$315 \times \frac{8}{\$9} = 280$$

Since $1.12\frac{1}{2} = \$1\frac{1}{8}$, or $\$\frac{9}{8}$, the number of yards is equal to the number of times $315 contains $\$\frac{9}{8}$, which may be found by dividing 315 by $\frac{9}{8}$, or by multiplying 315 by $\frac{8}{9}$.

Hence, 280 yards may be bought for $315.

Find the number of articles that can be bought for:

2. $65 at $1.25 each
3. $85 at $2.50 each
4. $28 at $3.50 each
5. $27 at $.37\frac{1}{2}$ each
6. $98 at $.66\frac{2}{3}$ each
7. $455 at $1.16\frac{2}{3}$ each
8. $540 at $2.25 each
9. $160 at $1.33\frac{1}{3}$ each

BILLS AND ACCOUNTS

185. The following shows a common form of *bill.*

CHICAGO, ILL., *Sept. 12,* 1906.

Mr. *John L. McLean,*
39 Roanoke Ave.

Bought of STONE BROS., 275 WATER ST.

DEALERS IN FRUITS OF ALL KINDS

TERMS: *Cash.* PHONE 147-D

		4 bu. Peaches	*1.65*	*6*	*60*	
		3 crates Muskmelons	*1.50*	*4*	*50*	
		12 baskets Grapes	*.18*	*2*	*16*	*13*	*26*
		Received payment,				
		Stone Bros.,				
		Per Ralph Stone.				

A bill properly made out should contain the following facts:

1. The name and address of the purchaser, or **debtor.**

2. The name and address of the person to whom the money is due, or the **creditor.**

3. The date when the debt was incurred.

4. A bill for goods sold should mention the articles sold, their number and price, the amount of each sale and the total amount, or **footing.** A bill for services rendered should show the nature of the services and the footing.

5. When a bill is paid, the words "Received payment," or "Paid," and the creditor's name, are written at the foot, either by the creditor or by some one authorized by him.

186. The following shows an **account.**

ALBANY, N.Y., *Oct. 1*, 1906.

Mr. C. L. Thomas, 180 Western Ave.

To EMPIRE FURNITURE CO., DR.

FURNITURE, CARPETS, OIL CLOTH, REFRIGERATORS, ETC.

TELEPHONE 946-A. 96 & 98 SOUTH PEARL STREET

			DR.		CR.	
Sept.	*1*	*Bed, Spring, & Mattress*	*20*	*00*		
		1 Washstand	*3*	*50*		
		1 Table	*3*	*50*		
		Cash			*15*	
Sept.	*15*	*1 Refrigerator*	*14*	*25*		
		Cash			*15*	
Oct.	*1*	*15 Stair Pads @ 12½*	*1*	*88*		
		1 Parlor Stove	*16*	*50*		
		Cash to balance			*29*	*63*
			59	*63*	*59*	*63*

187. The difference between the sum of the amounts owed and the sum of the amounts paid is called the **balance.**

WRITTEN EXERCISES

188. Make out, foot or balance, and receipt the following, supplying dates, names, and addresses, when lacking:

1. R. G. Palmer bought of Albert P. Hunt, 40 lb. sugar @ 6½¢; 1¼ lb. mixed spices @ 20¢; 5¼ lb. butter @ 32¢.

2. Chas. Sumner bought of Oscar Strauss 11 pieces of piano music @ 25¢; 79 pieces for orchestral instruments @ 10¢.

3. George A. Harris sold to Peter Harper, 3 doz. fruit jars @ 65¢; 25 lb. sugar @ 6¢; 1½ bu. tomatoes @ 60¢; ¼ lb. tea @ 65¢; 5 lb. coffee @ 35¢; 6 packages oatmeal @ 12½¢.

4. Jas. Osgood is debtor to Dr. Ellis Cook for 8 prescriptions @ $.50; 25 day visits @ $2; and 7 night visits @ $3.

5. Mar. 15, Pearl Roberts bought 1 encyclopedia @ $48 and 3 books @ $1.50, of the Haller Publishing Co. She paid $20 Mar. 15, $10 Apr. 15, $10 June 15, and the balance July 1.

6. Reuben Case is debtor to the Municipal Gas Co. for 6300 cu. ft. of gas for lighting @ 95¢ per 1000 cu. ft., and 11,200 cu. ft. of gas for fuel @ 80¢ per 1000 cu. ft.

7. 14 yd. silk @ $1.75; 2 pairs gloves @ $1.50; 9 yd. flannel @ 37½¢; 4 doz. handkerchiefs @ $2.75; 3 tablecloths @ $3.75; 1 rug at $32.50; 4 hassocks @ 75¢.

8. Porterhouse steak, 2¼ lb. @ 25¢; leg of veal, 14 lb. @ 15¢; pot roast, 5 lb. @ 12½¢; bacon, 6½ lb. @ 18¢; rib roast, 10¾ lb. @ 20¢; chickens, 8½ lb. @ 25¢.

9. Turkey, 11½ lb. @ 26¢; 2 qt. cranberries @ 12½¢; ½ pk. sweet potatoes @ 35¢; 1½ lb. sausage @ 16¢; 2½ qt. oysters @ 40¢; 1¾ gal. apple butter @ $1.25; 1½ doz. oranges @ 35¢.

10. 4 sets knives and forks @ $22.50; 4 sets fruit knives @ $4.85; 6 carving sets @ $7.50; 24 butcher knives @ $.50; 16 shears @ $.85; 60 pocket knives @ $.37½. Paid on account $75 at the time of purchase, and the balance 1 month later.

11. 24 baseballs @ $.87½; 1½ doz. baseball bats @ $.55 each; 4 catcher's mitts @ $4.75; 5 fielder's gloves @ $.95; 3 catcher's masks @ $2.25; 3 doz. tennis balls @ $4.50.

12. 3 shotguns @ $14.75; 12 shell bags @ 60¢; 3 shotgun covers @ $1.50; 12 hunting jackets @ $3.75; 16 pairs Indian clubs @ $.13; 6 basket balls @ $4.

PROBLEMS IN REVIEW

189. 1. Eva Crane, who lives in a suburb, decides to do her Christmas shopping in a city department store. She lives 15 minutes from the station. A train leaving at 8:37 A.M. reaches the city at 9:26 A.M. Thence it is 35 minutes to the store. How long does the entire journey take?

2. At the neckwear counter Eva buys 2 stocks at $37\frac{1}{2}¢$ each, 6 collars at $12\frac{1}{2}¢$ each, and 6 pieces of ruching, $\frac{3}{8}$ yd. each, at $24¢$ a yard. How much change should she receive from $10?

3. She buys a $4\frac{7}{8}$-yd. remnant of silk worth $72¢$ a yard for $3.15. How much does she save on the remnant?

The following articles Eva has charged to her mother's account. Find the cost of the purchases in each department.

4. In the toy department: a doll for $85¢$, 2 boxes of blocks @ $35¢$, a doll carriage for $1.25, and a toy motor car for $25¢$.

5. In the book department: "Alice in Wonderland" for $42¢$, "Little Women" and "Little Men," each at $95¢$, one Bible for $1.50, and Shakespeare's Works, 16 volumes, at $35¢$ each.

6. At the ribbon counter: $4\frac{3}{4}$ yd. ribbon at $36¢$ a yard, and 10 yd. baby ribbon at $1\frac{1}{2}¢$ a yard.

7. Grocery and meat department: $1\frac{1}{2}$ lb. nuts @ $20¢$, 2 lb. raisins @ $25¢$, 3 qt. cranberries @ $9¢$, an 8-lb. turkey @ $21\frac{1}{2}¢$.

8. Make out the bill that Mrs. Crane will receive on the first of the month for the articles that have been charged.

9. Mr. and Mrs. Clinton paid $93.68 for papering and painting the rooms of their new house, and $19.57 for staining the floors. How much did it cost them?

Find the cost of furnishings for:

10. PARLOR		**11.** HALL	
Rug	$30.00	Hat rack	9.25
Stand	9.00	Settle	16.50
Desk and chair	18.50	Book shelves	14.00
Rocking chairs	10.00	Stand	8.00
Morris chair	12.25	Turkish rug	35.40
Flemish chair	9.75	Rope portière	6.65
Curtains	25.00	Oak bookcase	18.00
Portières	21.50	Lace curtains	11.50
Pictures	72.00	Tabouret	2.35

12. Dining room: a rug, $18.60; a table, $16.00; 6 dining chairs @ $2.75; a table cover, $6.85; a sideboard, $12.25; and 3 pairs of window curtains @ $1.75.

13. Best bedroom: bed and mattress, $18; bolster roll, $2.75; dressing table, $10.45; cretonne bed set, $4; matting, 20 yd. @ 25¢; 2 pairs window curtains @ $.95; 2 chairs @ $1.65.

14. Nursery: matting, 17½ yd. @ 20¢; bed and mattress, $12; spread, $1.85; book shelves, $3; couch and cover, $10.00; rocking chair, $3.45; window curtains, $1.75; 8 pictures @ 75¢; 3 nursery posters @ 60¢.

15. Veranda: 3 porch chairs @ $1.50; hammock, $2.70; porch shades, $9; rug, $10.75.

16. Furnishing the kitchen, laundry, and cellar cost $49.85; 2 bedrooms, $56.75; bathroom, $15; upper hall and stairs, $19.35. Pictures not already mentioned cost $25. How much did it cost Mr. and Mrs. Clinton to decorate and furnish their whole house?

MEASUREMENTS *

EXERCISES

190. **1.** Give the table of *measures of length.*

2. How many inches are there in a yard? in $1\frac{1}{4}$ yd.?

3. What part of a yard is 18 in.? 27 in.?

4. How many feet are there in $1\frac{1}{2}$ yd.? What part of a foot is $\frac{1}{4}$ yd.? What part of a yard is $\frac{1}{2}$ ft.?

5. How many feet are there in 1 rd.? in 2 rd.? If a road is 66 ft. wide, how many rods wide is it?

6. How many rods are there in $\frac{1}{2}$ mi.? in $\frac{1}{4}$ mi.? in $1\frac{1}{2}$ mi.? What part of a mile is 40 rd.?

7. Give the table of *measures of surface.*

8. How many square inches are there in $\frac{1}{2}$ sq. ft.? What part of a square foot is 24 sq. in.? 60 sq. in.?

9. How many square feet are there in $2\frac{1}{2}$ sq. yd.? How many square yards are there in 189 sq. ft.?

10. How many acres are there in a plot of land 160 rd. long and 1 rd. wide? 320 rd., or 1 mile, long and 1 rd. wide? 1 mile long and 10 rd. wide? 1 mile long and 1 mile wide, or 1 mile square?

11. The area of a square, each side of which is a mile, is called a **square mile.**

1 square mile (sq. mi.) = 640 acres (A.).

A square mile of land is sometimes called a **section.**

12. How many acres are there in a quarter section?

13. Give the table of *measures of volume.*

14. How is the number of square inches in a square foot obtained? the number of cubic inches in a cubic foot?

* Tables will be found on pages 278–280.

15. Give the table of *liquid measures*.

16. How many pints are there in a gallon? in 1 gal. 2 qt.? in 3 gal. 1 qt.?

17. What part of a gallon is 1 qt.? 1 qt. 1 pt.? 2 qt. 1 pt.?

18. Give the table of *dry measures*.

19. How many quarts are there in $1\frac{1}{2}$ pk.? in $2\frac{1}{2}$ pk.? in 3 pk. 2 qt.? in 1 bu. 1 pk.? in 2 bu.?

20. Blackberries are shipped from North Carolina in bushel boxes, and in 2-bushel boxes. How many quarts does a bushel box hold? a 2-bushel box?

21. Strawberries are shipped from the South in crates of the following capacities: 24 qt., 32 qt., 36 qt., 42 qt., 45 qt., 48 qt., 60 qt. Express the different capacities in pecks.

How much less than 2 bushels does a 60-quart crate hold?

Express the capacity of a 48-quart crate in bushels.

22. The capacity of a refrigerator box for shipping Florida strawberries varies from 64 qt. to 128 qt. How many bushels does a 64-quart box hold? a 96-quart box? a 128-quart box?

23. Beans and other vegetables are often sold in $1\frac{1}{2}$-bushel boxes. How many pecks does such a box hold?

24. Tomatoes are often shipped in $\frac{1}{2}$-bushel crates. How much is such a crate of tomatoes worth at 10¢ per quart? How many quarts are there in a $\frac{5}{8}$-bushel crate?

25. Give the table of *time measures*.

26. Name the months and tell the number of days in each.

27. What part of a year is 2 months? 3 mo.? 9 mo.?

28. A cake of cement was kept under water for 32 months. How many years was it kept under water?

29. Alfred had a high fever for 54 hours. How many days and hours did the fever last?

30. If my watch gains 3 seconds a day, how many minutes will it gain during April?

31. Two watchmen relieved each other every 6 hours. How many times did each one watch during 6 days? How many hours did each watch during that time?

32. Jan. 1, 1906, came on Monday. Since one year = 52 weeks and 1 day, on what day of the week did Jan. 1, 1905, come?

Find the day of the week for Jan. 1, 1907.

33. What units of *weight* are used in weighing coal, iron, potatoes, meat, hay, etc.? Give the table.

34. How many ounces are there in $1\frac{1}{4}$ lb.? in $2\frac{3}{4}$ lb.?

35. Express in pounds 12 oz.; 20 oz.; 36 oz.; 50 oz.

36. How many pounds are there in $1\frac{1}{2}$ T.? in $2\frac{1}{4}$ T.?

37. A large block of marble weighed 14 tons. How many pounds did it weigh?

38. It takes 200 lb. of ice to keep a box of strawberries cool during shipment. What part of a ton does it take?

39. Find the cost of 3000 lb. of hay at \$9 per ton.

40. A man bought a ton of bran for \$14 and sold it at 90¢ per hundredweight. How much did he gain?

41. How many pounds are there in 2 T. 4 cwt. 75 lb.?

Counting

191. **1.** How many gloves make a *pair* of gloves?

2. How many persons make a *score* of persons?

3. How many oysters make a *dozen* oysters?

4. How many buttons are there in 12 dozen buttons, or in 1 *gross* of buttons?

5. How many pens are there in 12 boxes each of which contains 1 gross of pens? This number is a *great gross*.

6. Memorize this **counting table**:

2 = 1 pair
20 = 1 score
12 = 1 dozen
12 dozen = 144 = 1 gross
12 gross = 1728 = 1 great gross

WRITTEN EXERCISES

192. Find the gain from each purchase and sale:

	Articles	Quantity and Cost	Selling Price
1.	Pens	1 great gross @ $9.60	90¢ per gross
2.	Cuffs	1 doz. pairs @ $2.10	20¢ per pair
3.	Collars	20 doz. @ $1.60	15¢ each
4.	Hinges	1 doz. pairs @ $1.75	25¢ per pair
5.	Screws	100 gross @ 25¢	5¢ per doz.
6.	Pencils	4 gross @ $3	2 for 5¢
7.	Brackets	2 doz. pairs @ $7.50	80¢ per pair
8.	Door knobs	5 doz. pairs @ 66¢	6¢ per pair
9.	Coat hooks	18 gross @ $1.75	25¢ per doz.
10.	Indian clubs	30 pairs @ 28¢	35¢ per pair
11.	Rubber bands	25 gross @ 11¢	15¢ per gross
12.	Eggs	1 case (30 doz.), $5.10	20¢ per doz.
13.	Soap	1 box (240 cakes), $7.50	6 for 25¢
14.	Corn	1 case (24 cans), $1.70	3 for 25¢
15.	Beans	1 case (24 cans), $2.50	2 for 25¢
16.	Peaches	1 case (24 cans), $3.20	3 for 50¢
17.	Oranges	1 crate (96 oranges), $3	50¢ per doz.
18.	Oranges	1 crate (126 oranges), $2.80	40¢ per doz.

Stationers' Measures

193. Paper is sold in tablets by the 100, 200, 250, 500, and 1000 sheets, and by the pound or 100 pounds.

Folded sheets of paper are often sold by the following measures:

24 sheets = 1 quire
20 quires = 480 sheets = 1 ream

EXERCISES

194. 1. If a girl uses 1 doz. sheets of paper and 1 doz. envelopes per month, how many boxes of stationery, each containing 1 quire of paper and 24 envelopes, will she use in a year?

2. A boy buys a $2\frac{1}{2}$-quire box of writing paper. How many letters of 1 sheet each can he write on the paper?

3. Mr. Stahl bought a 5-quire box of writing paper and a "$\frac{1}{8}$-thousand" box of envelopes. How many more envelopes than sheets of paper did he buy?

4. Find the cost of 5 pounds of paper at 80¢ per pound and 500 envelopes at \$3.40 per thousand.

5. How many quires of foolscap paper are required to furnish 48 school children 2 sheets apiece during an examination?

Find the cost of the paper at \$1.60 per ream.

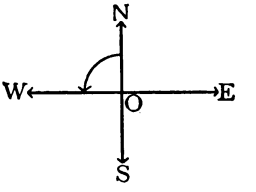
6. If legal cap that weighs 14 pounds to the ream costs 45¢ per $\frac{1}{4}$ ream, find the weight and cost of a quire.

7. The writing paper used by a boys' club cost 18¢ per quire. How much did it cost per sheet? per 100 sheets?

The envelopes cost \$3 per thousand. Find the cost of sending 100 invitations, each requiring a sheet of paper, an envelope, and a 2-cent stamp.

Measurement of Angles and Arcs

195. 1. Face toward the north, then turn until you face the west. Continue to turn until you face the north again.

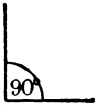


2. What part of a complete turn do you make in turning from north to west? from west to south? from south to east? from north to south?

3. Which is the greater amount of turning or change of direction — from north to northwest or from north to west? from north to west or from north to southwest?

196. That which expresses the amount of turning, or of change of direction, is called an **angle.**

197. A complete turn is called an angle of 360 **degrees.** Therefore $\frac{1}{360}$ of a complete turn is called 1 degree (1°).



RIGHT ANGLE

198. One fourth of a complete turn, or an angle of 90°, is called a **right angle.**

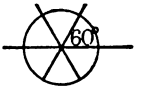
199. An angle that is less than a right angle is called an **acute angle.**

ACUTE ANGLE

200. An angle that is greater than a right angle but less than two right angles is called an **obtuse angle.**

OBTUSE ANGLE

201. The size of an angle does not depend on the length of the lines that form its *sides*, but only on the difference of direction of the sides, or on the amount of turning required to change the direction of one side to that of the other.

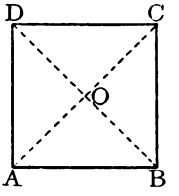


For example, these six angles are all equal, each being 60°, although the sides are of different lengths.

EXERCISES

202. **1.** Make a paper square. What kind of angles do the sides form? How many degrees are there in each?

2. Draw two lines connecting the opposite corners, as shown in the diagram.



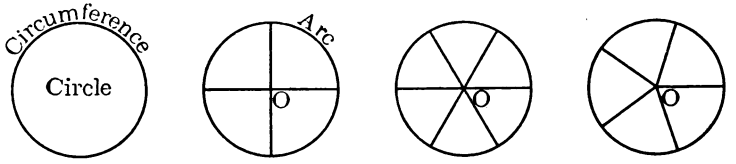
These lines are called **diagonals.**

3. Fold the paper square so that A lies on C. Crease it, then unfold it and notice whether the diagonal BD coincides with the crease.

How, then, are the right angles at B and D divided by the diagonal BD? How many degrees are there in each smaller angle at B and at D?

4. By folding the square again, find whether the angles at O are equal. How many degrees are there in each?

203. The boundary line of a circle is called its **circumference.** Any part of the circumference is called an **arc.**



204. **1.** In the second figure, what part of 360° is each angle? What part of the circumference is each arc?

2. What part of 360° is each angle and what part of the circumference is each arc in the third figure? in the fourth?

3. In these figures O, the *center* of each circle, is also the *vertex* of each angle. Such angles are called **central angles.**

4. Since an arc is the same part of a circumference that its central angle is of 360°, we measure *arcs* as well as *angles* in *degrees*, calling a circumference 360°.

5. How many degrees are there in an arc that is $\frac{1}{4}$ of a circumference? $\frac{1}{6}$ of a circumference? $\frac{1}{10}$ of a circumference?

6. What part of 360° is an angle or arc of 1 degree?

7. Angle degrees and arc degrees are divided into 60 parts called **minutes**, and minutes into 60 parts called **seconds**.

8. Memorize this table:

60 seconds (60″)	**= 1 minute (1′)**
60 minutes	**= 1 degree (1°)**
360 angle degrees	**= 4 right angles**
360 arc degrees	**= 1 circumference**

EXERCISES

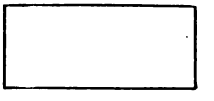
205. 1. How many degrees are there in $\frac{1}{3}$ of a circumference? in $\frac{1}{12}$ of a circumference? in 1200′? in 150′? in 2° 15′?

2. How many minutes are there in a right angle? in 5°? in 3° 40′? in 90″?

3. How many seconds are there in 40′? in 1°? in 10′ 20″? in 5′ 30″? in 2°? in $\frac{3}{4}$ of a minute?

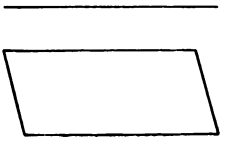
Measurement of Surfaces

206. A figure that has four straight sides and four right angles is called a **rectangle.**



If the sides of a rectangle are all equal, it is called a **square.**

207. Lines that cannot meet, however far they are extended, are called **parallel lines.**

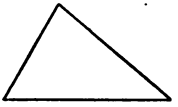


208. A four-sided figure whose opposite sides are parallel is called a **parallelogram.**

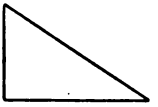
A rectangle is a right-angled parallelogram.

209. A three-sided figure is called a **triangle.**

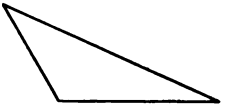
A triangle is *acute-angled* if all its angles are acute; *right-angled* if it has one right angle; *obtuse-angled* if it has one obtuse angle.



ACUTE-ANGLED TRIANGLE



RIGHT-ANGLED TRIANGLE

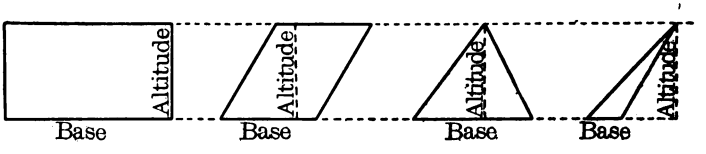


OBTUSE-ANGLED TRIANGLE

210. The two lines that form a right angle are said to be **perpendicular** to each other.

211. The side of a figure on which it is assumed to stand is called the **base,** and the height of the figure measured on a line perpendicular to the base is called the **altitude.**

The following figures have different bases but the same altitude.



212. Rectangles.

1. What is the area of a rectangle whose base is 24 in. and altitude 2 in.?

Find the area if the base is 2 ft. and the altitude is 2 in.

2. How is the area of a rectangle found?

What must be done if the base and altitude are expressed in different units?

3. Find the area of a rectangle whose base is 12 ft. and altitude 1 ft. How many times as great must the altitude be to make the area 48 sq. ft.?

4. The area of a certain rectangle is 48 sq. ft. If the base is 6 ft., what is the altitude?

If the base is 12 ft., what is the altitude?

If the altitude is 3 ft., what is the base?

213. If the base and altitude of a rectangle are expressed in *inches*, the area is found by multiplying the *number* of inches in the base by the *number* of inches in the altitude and calling the result *square inches*. If both dimensions are in *feet*, the area is found by multiplying one dimension by the other and calling the result *square feet*.

This is what is meant by the following statement:

The area of a rectangle is equal to the product of its base and altitude, expressed in like units.

It is seen that *either dimension is the quotient of the area and the other dimension, expressed in corresponding units.*

WRITTEN EXERCISES

214. 1. Find the area of a rectangle 12 ft. by 2 ft. 6 in.

SOLUTION

Base $= 12$ ft.
Alt. $= 2\frac{1}{2}$ ft.
Area $= 2\frac{1}{2} \times 12$ sq. ft. $= 30$ sq. ft.

Since the base and altitude must be expressed in *like units*, 2 ft. 6 in. is first changed to *feet*. Then the number of *square feet* in the area is found by multiplying 12 by $2\frac{1}{2}$.

2. If the area of a rectangular field is 24 acres and the length is 60 rods, what is the width?

SOLUTION

Area $= 24$ A. $= 24 \times 160$ sq. rd.
Length $= 60$ rd.

$$\text{Width} = \frac{24 \times 160}{60}\text{ rd.} = 64\text{ rd.}$$

Since the area and the length must be expressed in *corresponding units*, 24 A. is first changed to *square rods*. Then, the number of *rods* in the width is found by dividing 24×160 by 60.

The following incomplete table refers to various rectangles; complete the missing areas and dimensions:

	BASE	ALTITUDE	AREA
3.	21 ft., 6 in.	13 ft.	—— sq. ft.
4.	16.4 ft.	15 ft.	—— sq. ft.
5.	35 ft. 2 in.	12 ft.	—— sq. ft.
6.	——	7 in.	59.5 sq. in.
7.	18 yd.	——	252 sq. yd.
8.	$44\frac{3}{4}$ ft.	12 yd.	—— sq. ft.
9.	24 yd.	32 yd., 2 ft.	—— sq. yd.
10.	$12\frac{1}{2}$ yd.	60 ft.	—— sq. ft.
11.	85 rd.	——	42.5 Acres
12.	11 rd.	117 ft.	—— sq. ft.
13.	——	10 ft., 6 in.	6048 sq. in.

14. Draw a square whose area is 1 sq. ft. Draw a rectangle 16 in. long, having the same area as the square.

15. A rectangular farm containing 80 acres was 160 rods long. How wide was the farm?

16. A football field is 160 ft. wide, and its area is 52,800 sq. ft. Find the length of such a field.

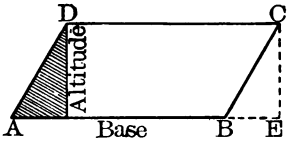
17. An acre is equal to 43,560 sq. ft. and a mile is equal to 5280 ft. If a gang plow throws 4 furrows, each 9 in. wide, how many miles must the plow travel to overturn an acre of land?

18. How many miles does a man walk in plowing 4 acres with a plow that throws a furrow 12 in. wide?

19. A dealer bought 5000 sheets of Imperial wove bond paper, 10 in. by 16 in., and made it into commercial note tablets, each containing 100 sheets 5 in. by 8 in. Find the number of tablets that he made.

215. Parallelograms.

1. If the shaded part of the parallelogram $ABCD$ is cut off and placed in the position BCE, what kind of a figure will be obtained?



2. Compare the base of the parallelogram with the base of the rectangle. Compare the altitudes. Compare the areas.

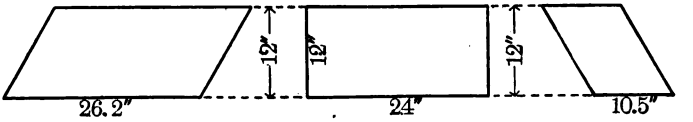
3. Draw any parallelogram. How does its area compare with that of a rectangle having the same base and altitude?

How, then, may the area of a parallelogram be found?

The area of a parallelogram is equal to the product of its base and altitude, expressed in like units.

WRITTEN EXERCISES

216. 1. Find the area of each of these parallelograms.



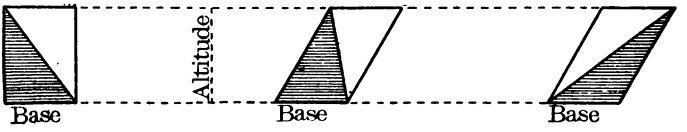
2. If the altitude of a parallelogram is 14 in. and the area is 308 sq. in., what is the base? Draw such a parallelogram.

Compute the missing areas and dimensions:

	BASE	ALTITUDE	AREA
3.	22 ft.	3 ft. 6 in.	—— sq. ft.
4.	42 in.	16 ft.	—— sq. ft.
5.	125.1 ft.	40 yd.	—— sq. ft.
6.	——	87 rd.	4263 sq. rd.
7.	4 rd.	88 ft.	—— sq. ft.
8.	160 rd.	—— rd.	95 A.

217. Triangles.

1. What part of each of these three parallelograms is the shaded triangle it contains?



2. How is the area of a parallelogram found? Then how is the area of a triangle found?

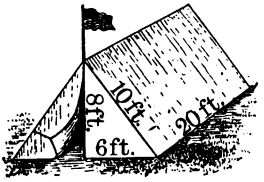
The area of a triangle is equal to half the product of its base and altitude, expressed in like units.

WRITTEN EXERCISES

218. Find the areas of triangles having these dimensions:

1. Base 25 in., altitude 18 in.
2. Base 42 ft., altitude 16 ft.
3. Base 33.4 ft., altitude 48 ft.
4. Base 106 ft. 3 in., altitude 60 ft.
5. Base 84 rd., altitude 80 rd. (Area in acres.)
6. Base 120 rd., altitude 72 rd.
7. Base 111.8 miles, altitude 96 miles.

8. The roof of this tower is composed of five triangular parts, each having a base of 7 ft. and an altitude of 9 ft. Find the area of the roof.



9. A man wishes to make a tent like this one, 20 ft. long, 12 ft. wide, 8 ft. high, and 10 ft. along the slanting edge. How many square yards of canvas does he need for each end? for each side? for the whole tent?

10. Find the area of the floor of the tent in square feet.

Measurement of Rectangular Solids

219. 1. If the part of the wall just begun is 12 ft. long, 2 ft. wide, and 1 ft. high, how many cubic feet of stone masonry has the mason laid?

2. How is this number of cubic feet found?

3. How many cubic feet of masonry will the wall contain when it is $4\frac{1}{2}$ times as high as it is now?

4. How is the volume of a *rectangular solid* 12 ft. long, 2 ft. wide, and $4\frac{1}{2}$ ft. high found?

The volume of a rectangular solid is equal to the product of its length, breadth, and thickness, all expressed in like units.

WRITTEN EXERCISES

220. Find the volume of rectangular solids having the following dimensions:

1. Length $19\frac{1}{4}$ ft., breadth 4 ft., thickness 2 ft.
2. Length 28 in., breadth 16 in., thickness $7\frac{1}{2}$ in.
3. Length 49 ft., breadth 14 ft., thickness 4 ft. 6 in.
4. Length 36 ft., breadth $2\frac{1}{2}$ yd., thickness 5 yd.
5. Length 22.75 ft., breadth 6 ft., thickness 4 ft.
6. Length $112\frac{1}{2}$ ft., breadth 5 ft., thickness 3 ft.
7. Find the volume of a stone sill 15 ft. long, 2 ft. wide, and 8 in. thick.
8. Find the volume of a cake of ice 3 ft. 2 in. long, 1 ft. 8 in. wide, and 14 in. thick.

PROBLEMS IN REVIEW

221. **1.** To measure distances on this map make a paper scale 3 in. long. Divide the first inch into 20 equal parts, as shown on the map. What distance does $\frac{1}{20}$ in. represent? $\frac{1}{5}$ in.?

Find to the nearest 25 miles the air-line distance between.

2. Omaha and Chicago.

3. Chicago and Boston.

4. St. Paul and Buffalo.

5. Portland and Baltimore.

6. Kansas City and Detroit.

7. Norfolk and New York.

8. Cleveland and Washington; Chicago and Philadelphia; Minneapolis and St. Louis; Indianapolis and Montreal.

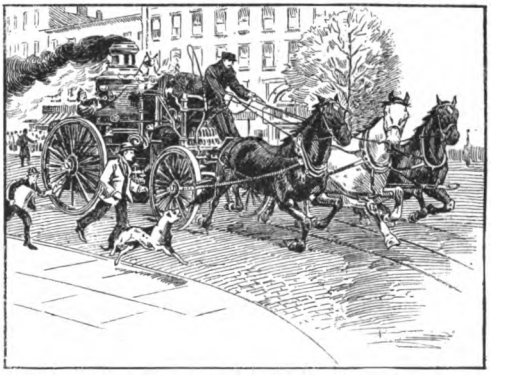
9. Find the approximate area of Kansas; of Pennsylvania.

10. How much wider is Ohio than Indiana?

11. The distance by rail from New York to Cleveland is 568 miles. How much less than this is the air-line distance?

12. The distance from Buffalo to Troy is 303 miles by rail, and 345 miles by the Erie Canal. Compare each of these distances with the air-line distance.

13. Ralph was examining the fire engine *Steamer 16* one day, when the fire alarm sounded. In 25 seconds the steamer and chemical wagon were out on the street; in 54 seconds more they had reached the scene of the fire. It took 28 seconds to connect with the hydrant; and 17 seconds later the steamer was throwing water upon the fire. How many minutes and seconds was it from the sounding of the alarm to the time when the water reached the fire?



14. The chemical wagon unreeled a 250-foot length of chemical hose and five 150-foot lengths of steamer hose. *Steamer 11* and *Chemical* then arrived with 500 feet of steamer hose and 250 feet of chemical hose. How much hose of each kind was used?

15. The first steamer pumped 900 gallons of water per minute for 1 hr. 40 min.; the second 500 gallons per minute for 1 hr. 19 min.; and a third steamer 750 gallons per minute for $1\frac{1}{2}$ hr. How much water was used to put out the fire?

16. Find the value of this water at \$.10 per 1000 gallons.

17. During the fire the three steamers used 1200 pounds of cannel coal costing \$18 per ton. Find the expense for fuel.

18. Between fires the water in the boiler of a fire engine is kept hot continually at an average cost of about 8¢ per day for fuel. Find the cost per year (365 days).

19. A hook-and-ladder truck weighs about 9600 pounds. Express the weight in tons.

FRACTIONS

Review

222. **1.** Write a fraction with figures.

Define a fraction; the terms of a fraction. What does the denominator show? the numerator?

2. Write seventeen eighteenths; twenty-five fortieths; sixty-five seventy-fifths; eleven one-hundred-twentieths.

3. Reduce $\frac{8}{12}$ to its lowest terms; to twenty-fourths.

When is a fraction expressed in its lowest terms? How may a fraction be reduced to higher terms?

4. Reduce 4 to eighths; $7\frac{3}{4}$ to fourths.

What is an integer? a mixed number? How may integers and mixed numbers be reduced to fractions?

5. Reduce $\frac{24}{8}$ to an integer; $\frac{16}{5}$ to a mixed number.

What is an improper fraction? a proper fraction?

How may an improper fraction be reduced to an integer or a mixed number? What is the value of a fraction?

6. Add $\frac{3}{8}$ and $\frac{7}{8}$. Subtract $\frac{2}{5}$ from $\frac{5}{6}$.

What are similar fractions? What must be done to fractions that are not similar before they can be added or subtracted?

7. Define reduction.

8. Multiply $\frac{3}{5}$ by 4; $\frac{3}{8}$ by 4.

How do you multiply a fraction by an integer?

9. Divide $\frac{4}{5}$ by 2; $\frac{3}{4}$ by 2.

Tell how to divide a fraction by an integer.

10. Multiply 6 by $\frac{2}{3}$. Divide 6 by $\frac{2}{3}$.

How do you multiply an integer by a fraction?

Tell how to divide an integer by a fraction. What is the reciprocal of a fraction?

EXERCISES

223. **1.** Reduce to lowest terms: $\frac{6}{8}$; $\frac{9}{12}$; $\frac{8}{10}$; $\frac{10}{12}$; $\frac{8}{14}$; $\frac{12}{16}$; $\frac{15}{20}$; $\frac{20}{24}$; $\frac{24}{32}$.

2. Reduce to an improper fraction: $4\frac{1}{2}$; $6\frac{2}{3}$; $9\frac{1}{4}$; $7\frac{5}{6}$; $8\frac{4}{5}$.

3. Reduce to an integer or a mixed number: $\frac{10}{2}$; $\frac{12}{5}$; $\frac{14}{4}$; $\frac{16}{8}$; $\frac{15}{3}$; $\frac{18}{5}$; $\frac{22}{6}$; $\frac{28}{8}$.

4. Reduce to fractions having the least common denominator: $\frac{3}{8}$ and $\frac{1}{4}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{5}{6}$ and $\frac{5}{8}$; $\frac{11}{12}$ and $\frac{7}{8}$.

Give answers promptly:

5. $\frac{3}{4}+\frac{2}{3}$	**13.** $\frac{3}{4}\div 4$	**21.** $\frac{4}{5}+\frac{3}{2}$	**29.** $8\times 2\frac{1}{2}$
6. $\frac{5}{6}-\frac{1}{2}$	**14.** $8\div\frac{1}{5}$	**22.** $\frac{8}{3}-\frac{3}{4}$	**30.** $6\frac{3}{4}\times 4$
7. $\frac{2}{3}$ of 9	**15.** $\frac{3}{5}+\frac{1}{4}$	**23.** $\frac{7}{4}\times 6$	**31.** $3\frac{1}{3}\div 2$
8. $\frac{3}{4}$ of 7	**16.** $4\div\frac{2}{3}$	**24.** $\frac{5}{2}\div 4$	**32.** $6\div 1\frac{1}{3}$
9. $5\times\frac{3}{8}$	**17.** $\frac{5}{8}-\frac{1}{3}$	**25.** $8\times\frac{5}{8}$	**33.** $3\frac{1}{2}-1\frac{7}{8}$
10. $\frac{4}{5}\times 6$	**18.** $8\times\frac{3}{4}$	**26.** $9\times\frac{6}{5}$	**34.** $1\frac{4}{5}+6\frac{3}{4}$
11. $\frac{6}{7}\div 2$	**19.** $6\times\frac{4}{5}$	**27.** $1\frac{2}{5}\div 4$	**35.** $5\frac{1}{4}+2\frac{3}{8}$
12. $\frac{1}{3}\div 8$	**20.** $\frac{5}{6}\div 2$	**28.** $1\frac{1}{4}\div 3$	**36.** $4\frac{2}{3}-3\frac{1}{2}$

37. There are 30 days in April and we received the daily paper on $\frac{5}{6}$ of them. How many papers did we receive?

38. If a newsboy made $\$\frac{1}{10}$ profit by selling some papers for $\$\frac{2}{5}$, how much did he pay for them?

39. Jane has gone to school $\frac{2}{3}$ as many years as Roy. She has attended for 4 years. How long has Roy attended?

40. When you study $4\frac{3}{4}$ hours a day in school and $2\frac{3}{8}$ hours at home, how long do you study each day?

41. How wide must I cut a strip of silk to make a ruffle $5\frac{1}{4}$ in. wide with a hem of $1\frac{3}{8}$ in. on one side and $\frac{3}{8}$ in. on the other?

42. How many yards of silk, 21 in. wide, will be needed to make this ruffle $5\frac{7}{8}$ yards long, allowing $\frac{1}{8}$ yd. for the seams?

43. When a steamship had slowed down to $\frac{1}{4}$ speed, it was going at the rate of 5 miles an hour. What was its rate at full speed?

44. When my pencil was new, it was $7\frac{1}{2}$ inches long. After I had used it a week, it was only $5\frac{3}{4}$ inches long. How much of it did I use in a week?

45. A certain locality had $4\frac{1}{3}$ inches of rainfall in one month and $2\frac{5}{8}$ inches the next. How much was the rainfall for both months?

46. A door is 8 feet high and its height is $2\frac{2}{3}$ times its width. How wide is it?

47. What is the area of the top of a table that is 10 feet long and $3\frac{1}{2}$ feet wide?

48. If a town has $9\frac{4}{5}$ miles of asphalt pavement and $5\frac{1}{2}$ miles of macadam, how much more asphalt than macadam has it?

WRITTEN EXERCISES

224. Reduce to lowest terms:

1. $\frac{44}{64}$	**3.** $\frac{84}{132}$	**5.** $\frac{125}{375}$	**7.** $\frac{216}{576}$	**9.** $\frac{330}{792}$
2. $\frac{36}{96}$	**4.** $\frac{72}{240}$	**6.** $\frac{192}{448}$	**8.** $\frac{288}{648}$	**10.** $\frac{625}{1000}$

11. Reduce to an improper fraction: $7\frac{5}{12}$; $8\frac{3}{10}$; $14\frac{3}{4}$; $19\frac{2}{3}$; $27\frac{4}{5}$; $32\frac{7}{8}$; $40\frac{5}{6}$; $51\frac{3}{5}$.

12. Reduce to a whole or a mixed number: $\frac{76}{2}$; $\frac{64}{3}$; $\frac{95}{4}$; $\frac{138}{6}$; $\frac{156}{8}$; $\frac{246}{10}$; $\frac{345}{12}$; $\frac{464}{16}$; $\frac{760}{24}$.

Do as the signs indicate:

13. $\frac{1}{3}+\frac{3}{4}+\frac{5}{8}$	**15.** $\frac{7}{8}-\frac{2}{3}+\frac{1}{4}$	**17.** $1\frac{2}{3}+3\frac{1}{4}-\frac{7}{8}$
14. $\frac{5}{6}+\frac{2}{3}+\frac{23}{24}$	**16.** $\frac{9}{16}+\frac{5}{8}-\frac{1}{2}$	**18.** $3\frac{1}{6}-1\frac{3}{8}+\frac{17}{24}$

First add and then subtract:

19. $43\frac{3}{4}$	**20.** $75\frac{4}{5}$	**21.** $94\frac{1}{6}$	**22.** $126\frac{3}{10}$	**23.** $291\frac{7}{10}$
$27\frac{5}{8}$	$42\frac{1}{3}$	$38\frac{7}{8}$	$46\frac{2}{5}$	$145\frac{5}{6}$

Add:

24. $21\frac{1}{3}$	**25.** $45\frac{1}{4}$	**26.** $88\frac{6}{7}$	**27.** $195\frac{3}{8}$	**28.** $765\frac{3}{4}$
$46\frac{3}{4}$	$17\frac{5}{6}$	$45\frac{3}{4}$	$346\frac{4}{5}$	$291\frac{5}{8}$
$16\frac{7}{8}$	$91\frac{2}{3}$	$24\frac{1}{2}$	$562\frac{1}{4}$	$452\frac{11}{12}$

Find in the shortest way:

29. $96 \times \frac{5}{6}$	**34.** $\frac{7}{12} \times 76$	**39.** $672\frac{3}{4} \div 36$
30. $84 \times \frac{7}{8}$	**35.** $\frac{15}{16} \div 10$	**40.** $548 \times 42\frac{2}{3}$
31. $\frac{2}{3} \div 24$	**36.** $419\frac{1}{6} \div 5$	**41.** $860 \div 68\frac{4}{7}$
32. $36 \div \frac{3}{5}$	**37.** $2653 \div \frac{7}{8}$	**42.** $\frac{5}{12}$ of 4298
33. $\frac{3}{4}$ of 62	**38.** $24 \times 563\frac{4}{5}$	**43.** $391\frac{7}{8} \times 124$

Find the cost of:

44. 25 plows at $\$6\frac{3}{4}$ each.

45. 18 horse rakes at $\$14\frac{9}{10}$ each.

46. 52 wagons at $\$46\frac{13}{20}$ each.

47. $16\frac{1}{2}$ tons of hay at \$13 per ton.

48. 612 bushels of corn at $\$\frac{5}{8}$ per bushel.

49. 1276 barrels of potatoes at $\$1\frac{7}{8}$ per barrel.

50. Find the annual cost of fuel to a manufacturer who uses 1800 tons of coal per year at $\$2\frac{3}{4}$ per ton.

51. A farmer put 1225 bushels of apples into barrels holding $2\frac{1}{2}$ bushels each. How many barrels were there?

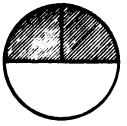
52. If a Hudson River steamboat goes 144 miles in $9\frac{3}{5}$ hours, what is its average rate per hour?

53. A farmer in the state of New York one year raised 948 bushels of grain on $16\frac{1}{2}$ acres of ground. What was the average yield per acre?

54. An automobile went 128 miles one day, $\frac{7}{8}$ as far the next, and $1\frac{1}{4}$ times as far the third day as the second. How far did it run in the three days?

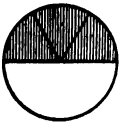
Multiplication of Fractions

225. Multiplication of fractions by fractions.



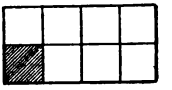
1. What part of a circle is $\frac{1}{2}$ of $\frac{1}{2}$ of it?

George had $\frac{1}{2}$ of a dollar, and he gave $\frac{1}{2}$ of his money to his sister. What part of a dollar did he give her?



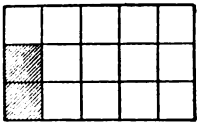
2. What part of a circle is $\frac{1}{3}$ of $\frac{1}{2}$ of it?

Emma cut $\frac{1}{2}$ of a pie into thirds and gave $\frac{1}{3}$ of $\frac{1}{2}$ to James. What part of the pie did she give him?



3. What part of a rectangle is $\frac{1}{2}$ of $\frac{1}{4}$ of it? $\frac{1}{4}$ of $\frac{1}{2}$ of it? $\frac{2}{4}$ of $\frac{1}{2}$ of it? $\frac{3}{4}$ of $\frac{1}{2}$ of it?

4. What part of a rectangle is $\frac{1}{3}$ of $\frac{1}{5}$ of it? $\frac{2}{3}$ of $\frac{1}{5}$ of it? $\frac{2}{3}$ of $\frac{2}{5}$ of it? $\frac{3}{3}$ of $\frac{1}{5}$ of it?



5. How does $\frac{1}{3}$ of $\frac{1}{5}$ compare in value with $\frac{1 \times 1}{3 \times 5}$? $\frac{2}{3}$ of $\frac{1}{5}$ with $\frac{2 \times 1}{3 \times 5}$? $\frac{2}{3}$ of $\frac{2}{5}$ with $\frac{2 \times 2}{3 \times 5}$? $\frac{3}{3}$ of $\frac{1}{5}$ with $\frac{2 \times 4}{3 \times 5}$?

6. Find the value of $\frac{1}{2}$ of $\frac{1}{3}$; $\frac{1}{2}$ of $\frac{2}{3}$; $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{1}{3}$ of $\frac{1}{4}$; $\frac{1}{4}$ of $\frac{1}{3}$; $\frac{3}{3}$ of $\frac{1}{4}$; $\frac{1}{4}$ of $\frac{3}{3}$; $\frac{1}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{1}{3}$; $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{3}{4}$ of $\frac{2}{3}$.

226. Finding a fractional part of a fraction is called **multiplying a fraction by a fraction.**

The word "of" between two fractions is equivalent to the sign $\times$, and when the sign $\times$ is used between two fractions, it may be read "of."

Such expressions are sometimes called **compound fractions.**

227. To multiply a fraction by a fraction, *multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.*

EXERCISES

228. Find quickly:

1. $\frac{1}{2}$ of $\frac{1}{6}$	6. $\frac{4}{5} \times \frac{1}{3}$	11. $\frac{3}{4}$ of $\frac{1}{6}$	16. $\frac{4}{5} \times \frac{5}{6}$
2. $\frac{1}{3}$ of $\frac{1}{3}$	7. $\frac{1}{2} \times \frac{2}{7}$	12. $\frac{2}{3}$ of $\frac{3}{5}$	17. $\frac{5}{8} \times \frac{2}{5}$
3. $\frac{1}{6}$ of $\frac{1}{4}$	8. $\frac{1}{4} \times \frac{4}{5}$	13. $\frac{1}{3}$ of $\frac{6}{7}$	18. $\frac{3}{4} \times \frac{5}{6}$
4. $\frac{1}{2}$ of $\frac{4}{5}$	9. $\frac{3}{5} \times \frac{1}{2}$	14. $\frac{5}{6}$ of $\frac{2}{3}$	19. $\frac{3}{5} \times \frac{3}{4}$
5. $\frac{1}{3}$ of $\frac{3}{8}$	10. $\frac{2}{3} \times \frac{1}{6}$	15. $\frac{2}{3}$ of $\frac{7}{8}$	20. $\frac{3}{4} \times \frac{4}{7}$

21. What is the area of the cover of a book, if it is $\frac{1}{2}$ of a foot long and $\frac{1}{3}$ of a foot wide?

22. The teacher lives $\frac{1}{3}$ of a mile from school and Bessie, $\frac{2}{5}$ as far. How far does Bessie have to walk to school?

23. The champion runner of our school circled a $\frac{1}{8}$-mile track in a minute. What distance did he run in $\frac{2}{3}$ of a minute?

24. It takes a single horse $\frac{6}{7}$ of a day to plow a field that a two-horse team can plow in $\frac{2}{5}$ of the time. In what part of a day can the team plow it?

25. If your father's garden plot contains $\frac{3}{4}$ of an acre and he gives you $\frac{1}{5}$ of it for your garden, what part of an acre does your garden contain?

WRITTEN EXERCISES

229. 1. Find $\frac{4}{5}$ of $\frac{7}{12}$, or multiply $\frac{7}{12}$ by $\frac{4}{5}$.

$$\frac{4}{5} \times \frac{7}{12} = \frac{4 \times 7}{5 \times 12} = \frac{7}{15}$$

To find $\frac{1}{5}$ of $\frac{7}{12}$ we divide $\frac{7}{12}$ by 5. You have learned that this may be done by multiplying the denominator by 5; then, $\frac{1}{5}$ of $\frac{7}{12} = \frac{7}{5 \times 12}$, and $\frac{4}{5}$ of $\frac{7}{12}$ = 4 times $\frac{1}{5}$ of $\frac{7}{12}$, or $\frac{4 \times 7}{5 \times 12}$.

Canceling and multiplying, the result is found to be $\frac{7}{15}$.

Or $\frac{\cancel{4}}{5} \times \frac{7}{\cancel{12}} = \frac{7}{15}$ (3)

It is not necessary to rewrite the fractions. We may simply cancel as in the second process.

Find:

2. $\frac{4}{5}$ of $\frac{5}{8}$
3. $\frac{5}{7}$ of $\frac{7}{9}$
4. $\frac{9}{8}$ of $\frac{5}{6}$
5. $\frac{6}{7}$ of $1\frac{4}{15}$
6. $\frac{4}{5}$ of $1\frac{5}{16}$
7. $\frac{5}{6}$ of $2\frac{4}{25}$
8. $2\frac{7}{28} \times \frac{7}{9}$
9. $1\frac{5}{32} \times \frac{4}{5}$
10. $2\frac{1}{40} \times \frac{8}{7}$
11. $\frac{5}{12} \times 1\frac{8}{25}$
12. $1\frac{5}{16} \times 3\frac{2}{35}$
13. $1\frac{0}{34} \times 1\frac{7}{50}$

14. Find the value of $\frac{2}{3}$ of $3\frac{3}{4}$.

SUGGESTION. — Reduce the mixed number to an improper fraction.

Find the value of:

15. $\frac{5}{8}$ of $10\frac{4}{5}$
16. $\frac{3}{4}$ of $13\frac{3}{5}$
17. $15\frac{5}{6} \times \frac{3}{5}$
18. $20\frac{1}{4} \times \frac{2}{9}$
19. $5\frac{2}{3} \times 7\frac{4}{5}$
20. $6\frac{7}{8} \times 4\frac{3}{10}$

21. Find the value of $3\frac{1}{3} \times \frac{1}{12} \times 3 \times 1\frac{3}{5}$.

$$3\tfrac{1}{3} \times \tfrac{1}{12} \times 3 \times 1\tfrac{3}{5} = \frac{\overset{2}{\cancel{10}}}{\underset{3}{\cancel{3}}} \times \frac{1}{\underset{3}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{1}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}}} = \frac{4}{3} = 1\tfrac{1}{3}$$

Reducing the mixed numbers to improper fractions, regarding the integer 3 as $\frac{3}{1}$, and canceling, the product is $\frac{4}{1}$, or $1\frac{1}{3}$.

Find the value of:

22. $1\frac{4}{7} \times \frac{3}{4} \times 14$
23. $6\frac{1}{4} \times 8 \times 4\frac{1}{5}$
24. $1\frac{1}{3} \times \frac{1}{10} \times 9 \times 5\frac{1}{6}$
25. $\frac{5}{6} \times \frac{1}{3} \times 12 \times 3\frac{3}{8}$
26. $1\frac{11}{15} \times \frac{5}{12} \times 4\frac{7}{8} \times 5\frac{1}{3}$
27. $2\frac{1}{2} \times \frac{3}{8} \times 9\frac{3}{5} \times \frac{3}{4} \times 1\frac{1}{3}$

28. How many feet are there in $\frac{5}{6}$ of a rod? in $2\frac{3}{4}$ rods?

29. John bought a book for $\$1\frac{3}{4}$ and afterward sold it for $\frac{4}{5}$ of the cost. How much did he receive for it?

30. At $\$6\frac{3}{4}$ per ton, find the cost of $5\frac{1}{2}$ tons of coal; of $11\frac{3}{5}$ tons.

31. Roscoe could dig potatoes $\frac{2}{3}$ as fast as John. How many bushels could he dig while John was digging $42\frac{3}{4}$ bushels?

32. Find the area of a board walk that is $24\frac{2}{3}$ feet long and $3\frac{1}{2}$ feet wide.

33. During a feeding test a lamb gained $40\frac{1}{2}$ pounds, at an average cost of $2\frac{8}{9}$ cents per pound. Find the total cost.

34. How many cubic feet of coal will a bin hold, if it is $12\frac{1}{2}$ feet long, $8\frac{1}{4}$ feet wide, and $6\frac{2}{3}$ feet deep?

35. If butter fat will make $1\frac{1}{6}$ times its weight of butter and milk is $\frac{1}{24}$ butter fat, how many pounds of butter can be made from 720 pounds of milk?

Division of Fractions

230. Division of fractions by fractions.

1. How many toys can be bought for $\$\frac{4}{5}$ at $\$\frac{1}{5}$ each? at $\$\frac{2}{5}$?

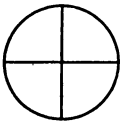
2. How many cups can be filled out of $\frac{6}{8}$ gal., if each holds $\frac{1}{8}$ gal.? $\frac{2}{8}$ gal.? $\frac{3}{8}$ gal.? As the divisor grows larger what happens to the quotient?

3. $\frac{4}{5} \div \frac{1}{5} = ?$ $\frac{4}{5} \div \frac{2}{5} = ?$ $\frac{6}{8} \div \frac{2}{8} = ?$ $\frac{6}{8} \div \frac{3}{8} = ?$

How may we divide one fraction by another when the fractions are similar?

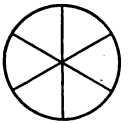
4. How many times is $\frac{1}{4}$ contained in 1? in $\frac{1}{2}$?

What part of the first result is the second?



5. How many times is $\frac{1}{6}$ contained in 1? in $\frac{1}{2}$? in $\frac{1}{3}$? in $\frac{2}{3}$?

What part of the first result is the second? the third? the fourth?



6. Divide 1 by $\frac{5}{6}$ and from the result find $\frac{1}{2} \div \frac{5}{6}$; $\frac{1}{3} \div \frac{5}{6}$; $\frac{2}{3} \div \frac{5}{6}$.

7. Compare $\frac{1}{2} \div \frac{5}{6}$ with $\frac{1}{2} \times \frac{6}{5}$; $\frac{2}{3} \div \frac{5}{6}$ with $\frac{2}{3} \times \frac{6}{5}$; $\frac{4}{5} \div \frac{2}{5}$ with $\frac{4}{5}$ multiplied by the *reciprocal* of $\frac{2}{5}$.

231. To divide a fraction by a fraction, *change to similar fractions and divide the numerator of the dividend by the numerator of the divisor.* Or,

Multiply the dividend by the reciprocal of the divisor.

EXERCISES

232. Give quotients quickly:

1. $\frac{1}{2} \div \frac{1}{4}$	**6.** $\frac{1}{2} \div \frac{2}{3}$	**11.** $\frac{4}{5} \div \frac{1}{2}$	**16.** $\frac{7}{10} \div \frac{1}{2}$
2. $\frac{2}{3} \div \frac{1}{6}$	**7.** $\frac{5}{6} \div \frac{1}{2}$	**12.** $\frac{3}{4} \div \frac{2}{3}$	**17.** $\frac{9}{10} \div \frac{2}{5}$
3. $\frac{3}{4} \div \frac{1}{8}$	**8.** $\frac{7}{8} \div \frac{1}{4}$	**13.** $\frac{1}{3} \div \frac{2}{5}$	**18.** $\frac{5}{12} \div \frac{2}{3}$
4. $\frac{3}{8} \div \frac{1}{4}$	**9.** $\frac{5}{6} \div \frac{2}{3}$	**14.** $\frac{2}{3} \div \frac{3}{4}$	**19.** $\frac{7}{12} \div \frac{1}{4}$
5. $\frac{3}{4} \div \frac{1}{2}$	**10.** $\frac{3}{8} \div \frac{3}{4}$	**15.** $\frac{3}{4} \div \frac{5}{6}$	**20.** $\frac{11}{12} \div \frac{3}{4}$

21. Mary's mother gave her $\$\frac{3}{4}$ with which to buy ribbon at $\$\frac{1}{4}$ a yard. How many yards could she buy?

22. At $\$\frac{1}{8}$ per pound, how many pounds of copper can be bought for $\$\frac{1}{2}$?

23. How long will $\frac{2}{3}$ of a barrel of flour last a family that uses $\frac{1}{6}$ of a barrel a week?

24. A man took a bicycle trip, covering $\frac{1}{9}$ of the distance each hour. How long had he been riding when he had covered $\frac{2}{3}$ of the distance?

25. A coal dealer wishes to put $\frac{4}{5}$ of a ton of coal into bags that hold $\frac{1}{20}$ of a ton each. How many bags does he need?

WRITTEN EXERCISES

233. **1.** Divide $\frac{15}{16}$ by $\frac{5}{6}$.

$$\frac{15}{16} \div \frac{5}{6} = \frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{16}}} \times \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{5}}} = \frac{9}{8} = 1\frac{1}{8}$$

Since $1 \div \frac{5}{6}$ is $\frac{6}{5}$, $\frac{15}{16} \div \frac{5}{6}$ is $\frac{15}{16}$ of $\frac{6}{5}$, or $\frac{15}{16} \times \frac{6}{5}$. Canceling and multiplying, the result is found to be $\frac{9}{8}$, or $1\frac{1}{8}$.

Divide:

2. $\frac{9}{10}$ by $\frac{5}{8}$	**5.** $\frac{17}{18}$ by $\frac{1}{6}$	**8.** $\frac{5}{6}$ by $\frac{3}{14}$	**11.** $\frac{17}{20}$ by $\frac{5}{12}$
3. $\frac{7}{12}$ by $\frac{4}{9}$	**6.** $\frac{14}{15}$ by $\frac{7}{8}$	**9.** $\frac{7}{8}$ by $\frac{5}{24}$	**12.** $\frac{23}{32}$ by $\frac{3}{16}$
4. $\frac{11}{16}$ by $\frac{3}{4}$	**7.** $\frac{24}{25}$ by $\frac{4}{5}$	**10.** $\frac{8}{9}$ by $\frac{4}{35}$	**13.** $\frac{35}{48}$ by $\frac{7}{24}$

14. Divide $4\frac{3}{4}$ by $\frac{5}{8}$.

SUGGESTION. — Reduce the mixed number to an improper fraction.

Divide:

15. $5\frac{1}{2}$ by $\frac{3}{4}$	**19.** $12\frac{1}{4}$ by $1\frac{2}{5}$	**23.** $28\frac{3}{4}$ by $12\frac{1}{2}$
16. $7\frac{2}{3}$ by $\frac{5}{6}$	**20.** $16\frac{1}{5}$ by $6\frac{3}{4}$	**24.** $44\frac{2}{5}$ by $13\frac{7}{8}$
17. $9\frac{4}{5}$ by $\frac{7}{8}$	**21.** $18\frac{1}{6}$ by $5\frac{5}{8}$	**25.** $270\frac{5}{6}$ by $31\frac{1}{4}$
18. $6\frac{3}{8}$ by $\frac{2}{3}$	**22.** $25\frac{2}{3}$ by $4\frac{5}{7}$	**26.** $462\frac{6}{7}$ by $64\frac{4}{5}$

27. Find the value of $1\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{3}{8} \div 3 \div \frac{1}{6}$.

$$1\tfrac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{3}{8} \div 3 \div \frac{1}{6} = \frac{5}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{8}{3} \times \frac{1}{\cancel{3}} \times \frac{\cancel{6}^{2}}{1} = \frac{16}{3} = 5\tfrac{1}{3}$$

After reducing all integers and mixed numbers to improper fractions, we take the reciprocals of the fractions that are divisors and write them as multipliers. Canceling and reducing, the result is $5\frac{1}{3}$.

Find the value of:

28. $\frac{3}{8} \times \frac{5}{6} \div \frac{2}{3} \div \frac{3}{4}$	**31.** $\frac{7}{10} \times \frac{5}{8} \div 2\frac{1}{2} \div 4 \div \frac{1}{5}$
29. $\frac{1}{6} \times 7 \div \frac{4}{5} \div \frac{7}{9}$	**32.** $\frac{5}{12} \div 1\frac{1}{3} \div \frac{5}{9} \times \frac{1}{8} \times \frac{5}{6}$
30. $1\frac{2}{3} \div \frac{5}{6} \times \frac{1}{2} \div \frac{6}{7}$	**33.** $\frac{14}{15} \times \frac{2}{7} \div \frac{1}{3} \times \frac{5}{8} \div \frac{3}{4}$

34. Divide $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ by $\frac{1}{4}$ of $\frac{5}{12}$.

SUGGESTION. — $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ divided by $\frac{1}{4}$ of $\frac{5}{12} = \frac{1}{2} \times \frac{3}{8} \times \frac{5}{6} \times \frac{4}{1} \times \frac{12}{5}$.

Divide:

35. $\frac{2}{3}$ of $\frac{5}{7}$ of $2\frac{1}{2}$ by $\frac{3}{8}$ of $\frac{2}{11}$ of $\frac{4}{5}$

36. $\frac{5}{6}$ of $\frac{13}{14}$ of $\frac{7}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{3}{4}$ of 26

37. $\frac{7}{12}$ of $2\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{7}{10}$ of $\frac{1}{8}$ of $3\frac{2}{3}$

38. $\frac{3}{4}$ of $1\frac{5}{8}$ of $4\frac{1}{5}$ by $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{9}{10}$ of $6\frac{1}{4}$

39. $\frac{7}{8}$ of $1\frac{1}{12}$ of $\frac{4}{5}$ of 75 by $\frac{1}{4}$ of $\frac{9}{16}$ of $\frac{5}{6}$ of $1\frac{1}{24}$ of 32

40. How much candy can you buy for $\$1\frac{1}{2}$ at $\$\frac{2}{5}$ per pound?

41. A street vender has $2\frac{3}{4}$ pecks of chestnuts. How many times will they fill a measure that holds $\frac{1}{8}$ of a peck?

42. How many times can $5\frac{1}{4}$ gallons of vinegar be drawn from a barrel that contains $31\frac{1}{2}$ gallons?

43. A farmer received $\$238\frac{3}{4}$ for some corn at $\$\frac{5}{8}$ per bushel. How many bushels did he sell?

44. A certain kind of barbed wire weighs $1\frac{3}{8}$ lb. per rod. How much does it weigh per foot?

45. If it takes $18\frac{5}{6}$ yards of canvas to make a tent, how many tents can be made out of $150\frac{2}{3}$ yards?

46. If it requires $44\frac{1}{3}$ cubic yards of earth to level the lawn in front of Dr. Robinson's house, how many loads of $1\frac{7}{12}$ cubic yards will be needed?

47. If a corn-fed hog gains $11\frac{3}{4}$ pounds for each bushel of corn that he eats, how many bushels will increase his weight $70\frac{1}{2}$ pounds?

48. An experiment in feeding wheat showed that a hog gained $13\frac{1}{2}$ pounds for each bushel of wheat fed. How many bushels of wheat were required to increase his weight from $225\frac{3}{4}$ pounds to $397\frac{7}{8}$ pounds?

234. Simplifying complex fractions.

You have learned that fractions indicate division, the numerator being the dividend and the denominator the divisor.

We may indicate the division of a fraction by an integer, of an integer by a fraction, or of a fraction by a fraction, in *fractional form*, by writing the dividend above a line and the divisor below.

We may write, $\frac{2}{3} \div 6$ like this, $\frac{\frac{2}{3}}{6}$; $8 \div \frac{3}{4}$ like this, $\frac{8}{\frac{3}{4}}$; and $\frac{1}{2} \div \frac{4}{5}$ like this, $\frac{\frac{1}{2}}{\frac{4}{5}}$.

Such indicated expressions of division are sometimes called **complex fractions.**

When the indicated division is performed, the complex fraction is said to be **simplified.**

WRITTEN EXERCISES

235. **1.** Simplify $\frac{\frac{3}{4}}{\frac{5}{6}}$.

SOLUTION. $\frac{\frac{3}{4}}{\frac{5}{6}} = \frac{3}{4} \div \frac{5}{6} = \frac{3}{\cancel{4}_{2}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{9}{10}$

Simplify:

2. $\frac{\frac{5}{8}}{\frac{3}{4}}$	**6.** $\frac{8}{\frac{4}{5}}$	**10.** $\frac{1\frac{3}{4}}{1\frac{1}{2}}$	**14.** $\frac{4\frac{7}{8}}{19\frac{1}{2}}$	**18.** $\frac{3\frac{1}{4}+\frac{2}{3}}{6\frac{3}{4}-\frac{7}{8}}$
3. $\frac{\frac{6}{7}}{\frac{2}{5}}$	**7.** $\frac{\frac{3}{8}}{6}$	**11.** $\frac{2\frac{1}{2}}{3\frac{1}{3}}$	**15.** $\frac{8\frac{1}{3}}{33\frac{1}{3}}$	**19.** $\frac{\frac{2}{3}\text{ of }8}{6\frac{2}{3}}$
4. $\frac{\frac{7}{8}}{\frac{7}{12}}$	**8.** $\frac{6}{3\frac{1}{3}}$	**12.** $\frac{3\frac{3}{4}}{6\frac{2}{3}}$	**16.** $\frac{6\frac{1}{4}}{37\frac{1}{2}}$	**20.** $\frac{2\frac{9}{10}}{\frac{1}{2}\text{ of }\frac{4}{5}}$
5. $\frac{\frac{5}{6}}{\frac{15}{16}}$	**9.** $\frac{5\frac{1}{2}}{4}$	**13.** $\frac{5\frac{1}{5}}{5\frac{4}{7}}$	**17.** $\frac{62\frac{1}{2}}{66\frac{2}{3}}$	**21.** $\frac{7}{8}\text{ of }\frac{5}{4\frac{3}{8}}$

236. Finding what part one number is of another.

1. What part of 4 is 3? of 6 is 2? of 12 is 8?

2. What part of \$8 is \$4? of 3 pints is 2 pints? of 4 fifths is 2 fifths? of $\frac{4}{5}$ is $\frac{2}{5}$? of $\frac{5}{6}$ is $\frac{1}{6}$? of $\frac{6}{7}$ is $\frac{4}{7}$?

3. What part of 1 pound is 8 ounces? of 4 yards, or 12 feet, is 6 feet? of $\frac{1}{2}$, or $\frac{2}{4}$, is $\frac{1}{4}$? of $\frac{5}{8}$ is $\frac{1}{4}$, or $\frac{2}{8}$?

4. Tell how to find what part one number is of another.

WRITTEN EXERCISES

237. **1.** What part of 96 is 64? of $3\frac{1}{2}$ is $\frac{7}{8}$?

$\frac{64}{96} = \frac{2}{3}$ $\qquad$ $\frac{\frac{7}{8}}{3\frac{1}{2}} = \frac{7}{8} \div 3\frac{1}{2} = \frac{7}{8} \div \frac{7}{2} = \frac{\overset{1}{\cancel{7}}}{\underset{4}{\cancel{8}}} \times \frac{2}{\cancel{7}_{1}} = \frac{1}{4}$

2. What part of 144 is 80? of $\frac{5}{12}$ is $\frac{2}{3}$? of 12 is $\frac{3}{8}$?

What part of

3. 225 is 75?	**8.** $\frac{5}{6}$ is $\frac{3}{8}$?	**13.** $8\frac{1}{3}$ is $6\frac{2}{3}$?
4. 896 is 280?	**9.** $\frac{8}{9}$ is $\frac{4}{5}$?	**14.** $5\frac{5}{8}$ is $3\frac{3}{4}$?
5. 1000 is 875?	**10.** $\frac{15}{16}$ is $\frac{5}{8}$?	**15.** $12\frac{3}{4}$ is $6\frac{4}{5}$?
6. 1728 is 576?	**11.** 25 is $2\frac{1}{2}$?	**16.** $31\frac{1}{4}$ is $9\frac{3}{8}$?
7. 2240 is 1344?	**12.** 75 is $6\frac{1}{4}$?	**17.** $66\frac{2}{3}$ is $37\frac{1}{2}$?

18. When an acre of land yields $18\frac{3}{4}$ bushels of grain, what part of an acre will yield $12\frac{1}{2}$ bushels?

19. If $26\frac{2}{3}$ pounds of cream produced $6\frac{2}{3}$ pounds of butter fat, what part of the cream was butter fat?

20. The celery from an acre of land sold for $\$218\frac{3}{4}$ and the cost of raising it was $\$87\frac{1}{2}$. What part of the selling price was the cost?

21. The height of the Statue of Liberty is about 150 feet, and the length of the forefinger holding the torch $8\frac{1}{3}$ feet. What part of the whole height is the length of the forefinger?

22. A public park has an area of $325\frac{2}{5}$ acres, of which $108\frac{7}{15}$ acres are occupied by a lake. What fractional part of the park does the lake occupy?

238. Comparison of fractions.

1. Compare $\frac{2}{3}$ with $\frac{1}{3}$ in this way: $\frac{2}{3} \div \frac{1}{3} = 2$; that is, $\frac{2}{3}$ is 2 times $\frac{1}{3}$.

In the same way compare $\frac{4}{5}$ with $\frac{2}{5}$; $\frac{1}{2}$ with $\frac{1}{4}$; $\frac{1}{2}$ with $\frac{1}{8}$; $\frac{1}{3}$ with $\frac{1}{6}$; $\frac{2}{3}$ with $\frac{1}{6}$; $\frac{3}{4}$ with $\frac{1}{2}$.

2. Compare $\frac{1}{3}$ with $\frac{2}{3}$ in this way: $\frac{1}{3} \div \frac{2}{3} = \frac{1}{2}$; that is, $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$.

In the same way compare $\frac{2}{5}$ with $\frac{4}{5}$; $\frac{1}{4}$ with $\frac{1}{2}$; $\frac{1}{8}$ with $\frac{1}{2}$; $\frac{1}{6}$ with $\frac{1}{3}$; $\frac{1}{6}$ with $\frac{2}{3}$; $\frac{1}{2}$ with $\frac{3}{4}$.

3. Compare $\frac{1}{2}$ with $\frac{1}{6}$; $\frac{1}{6}$ with $\frac{1}{2}$; $\frac{5}{6}$ with $\frac{2}{3}$; $\frac{2}{3}$ with $\frac{5}{6}$; $\frac{1}{2}$ with $\frac{1}{3}$; $\frac{1}{3}$ with $\frac{1}{2}$; $1\frac{1}{4}$ with $\frac{1}{4}$; $\frac{1}{4}$ with $1\frac{1}{4}$.

EXERCISES

239. Compare:

1. $\frac{3}{4}$ with $\frac{3}{8}$
2. $\frac{3}{8}$ with $\frac{3}{4}$
3. $\frac{1}{3}$ with $\frac{1}{2}$
4. $\frac{1}{2}$ with $\frac{2}{3}$
5. $\frac{1}{2}$ with $\frac{2}{5}$
6. $\frac{1}{4}$ with $\frac{1}{6}$
7. $\frac{3}{4}$ with $\frac{5}{6}$
8. $\frac{5}{8}$ with $\frac{1}{4}$
9. $1\frac{1}{2}$ with $\frac{1}{2}$
10. $2\frac{3}{4}$ with $\frac{1}{4}$
11. $1\frac{1}{3}$ with $\frac{2}{3}$
12. $2\frac{2}{3}$ with $\frac{1}{3}$
13. $\frac{2}{3}$ with $1\frac{2}{3}$
14. $\frac{5}{8}$ with $1\frac{1}{4}$
15. $\frac{3}{4}$ with $2\frac{1}{2}$
16. $\frac{5}{6}$ with $3\frac{1}{3}$

17. Compare $1\frac{1}{2}$ with $\frac{1}{4}$. When $\frac{1}{4}$ of a dozen bananas costs 5 cents, find the cost of $1\frac{1}{2}$ dozen.

18. Compare $\frac{1}{4}$ with $1\frac{1}{4}$. When 15 quarts of strawberries cost $\$1\frac{1}{4}$, how many quarts can be bought for $\$\frac{1}{4}$?

19. If my watch gains $\frac{1}{6}$ of a minute in 3 days, in how many days will it gain $1\frac{1}{3}$ minutes?

20. If 24 bunches of celery cost $\$2\frac{2}{5}$, how many bunches can be bought for $\$\frac{3}{5}$?

21. If 2 pounds of live cocoons make $\frac{1}{4}$ of a pound of silk, how many pounds of cocoons are required for $\frac{7}{8}$ of a pound of silk?

22. John walked $2\frac{1}{4}$ miles in $\frac{3}{4}$ of an hour. At that rate, how long would it take him to walk $1\frac{1}{2}$ miles?

WRITTEN EXERCISES

240. Compare:

1. $\frac{9}{10}$ with $\frac{3}{4}$
2. $\frac{5}{12}$ with $\frac{2}{3}$
3. $\frac{15}{16}$ with $\frac{5}{6}$
4. $\frac{11}{18}$ with $\frac{2}{3}$
5. 24 with $6\frac{2}{3}$
6. 88 with $5\frac{3}{5}$
7. $8\frac{1}{4}$ with 66
8. $6\frac{1}{6}$ with 75
9. $87\frac{1}{2}$ with $8\frac{1}{3}$
10. $15\frac{3}{4}$ with $94\frac{1}{2}$
11. $46\frac{2}{3}$ with $6\frac{5}{12}$
12. $91\frac{1}{3}$ with $22\frac{5}{6}$

13. Compare $\frac{2}{3}$ with $\frac{11}{12}$. If $\frac{11}{12}$ of a barrel of flour makes 231 loaves of bread, how many loaves will $\frac{2}{3}$ of a barrel make?

14. When 18 barrels of cabbages can be bought for $\$22\frac{1}{2}$, how many barrels can be bought for $\$7\frac{1}{2}$?

15. Compare $3\frac{3}{4}$ with $\frac{3}{5}$. If a woman receives $\$\frac{3}{5}$ for picking 40 quarts of strawberries, how many quarts must she pick to earn $\$3\frac{3}{4}$?

16. A farmer raised $6\frac{2}{3}$ tons of hay on 4 acres. If his whole crop was $86\frac{1}{4}$ tons, how many acres of hay had he?

17. If an automobile runs 8 miles while a train runs $26\frac{1}{4}$ miles, how far does it go while the train runs $7\frac{7}{8}$ miles?

18. When the cost of transporting 90 cans of milk from Medina, Ohio, to Cleveland is $\$13\frac{1}{2}$, how many can be transported for $\$1\frac{1}{5}$?

19. If it takes $3\frac{3}{4}$ bushels of seed to sow $4\frac{1}{2}$ acres of buckwheat, how many bushels are needed to sow $28\frac{4}{5}$ acres?

20. An orchard yielded $427\frac{1}{2}$ barrels of apples, averaging $31\frac{2}{3}$ barrels to 10 trees. Find the number of trees in the orchard.

MISCELLANEOUS EXERCISES

241. 1. Find the cost of $4\frac{3}{8}$ yards of linen at $\$\frac{2}{5}$ per yard.

2. At a certain gold mine it costs $\$\frac{3}{5}$ a ton to mine the ore. How many tons of gold ore can be mined for $\$61\frac{1}{2}$? Find the cost of crushing this quantity of ore at $\$\frac{9}{10}$ per ton.

3. A train running from Boston to Providence averages $6\frac{1}{8}$ miles every 7 minutes. What is its rate per hour? How long does it take the train to go from one city to the other, the distance being 49 miles?

4. Estimating $3\frac{1}{2}$ barrels of oil equivalent to one ton of coal, a certain Texas railroad would have needed 910,938 barrels of oil to run its trains one year. How much coal was used?

5. A woman in Ceylon picks 16 pounds of tea leaves in one day from full-grown plants, but only $7\frac{1}{2}$ pounds from young plants. At $\frac{3}{4}¢$ per pound, how much more does she earn per day on old plants?

6. It took $6\frac{1}{4}$ yards of cloth at $\$\frac{1}{2}$ per yard for Elsie's dress, and $11\frac{1}{4}$ yards at $\$\frac{3}{5}$ per yard for her mother's. How much less did the cloth for Elsie's dress cost than for her mother's?

7. A boy bought 90 cocoanuts at $\$\frac{1}{2}$ per dozen, and sold them at $\$\frac{1}{20}$ apiece. How much money did he gain?

8. A grocer bought a bunch of bananas for $\$1\frac{1}{4}$. He sold $3\frac{1}{3}$ dozen from the top at $\$\frac{3}{20}$ per dozen, and the rest, $6\frac{1}{4}$ dozen, at $\$\frac{1}{5}$ per dozen. How much did he gain?

9. If a 224-pound bag of salt costs $\$1\frac{3}{5}$, how much will a 56-pound bag cost at the same rate?

10. When 16 pounds of salt cost $\$\frac{1}{10}$, how many pounds are there in a bag that costs $\$1\frac{2}{5}$?

11. The cost of oiling a stretch of highway in Pennsylvania was \$65,000, of which the state paid $\$43{,}333\frac{1}{3}$. What part of the cost was paid by the state?

12. Mr. Thayer owned $\frac{3}{4}$ of a section of land. He sold $\frac{5}{8}$ of his land to Mr. Hall, who gave $\frac{2}{5}$ of his part to a son. What part of the whole section did Mr. Hall's son receive? How many acres?

13. In constructing $9\frac{3}{16}$ miles of railroad track 2640 ties were used per mile. Find the whole number of ties used and their cost at $\$\frac{4}{5}$ each.

14. If $57\frac{1}{6}$ bushels of seed are required for a rice field of $24\frac{1}{2}$ acres in South Carolina, and $\frac{5}{6}$ of a bushel for $\frac{2}{3}$ of an acre in Japan, how much seed is required per acre in each place? how much more per acre in South Carolina than in Japan?

15. A grain elevator had a bin $7\frac{1}{8}$ ft. square and 80 ft. deep. How many bushels would it hold, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

16. A solid board fence 45 yd. long and $1\frac{7}{8}$ yd. high was painted two coats. It required $18\frac{3}{4}$ lb. of paint for the first coat and $13\frac{1}{2}$ lb. for the second. How many square yards did a pound of paint cover for the first coat? for the second?

DECIMAL FRACTIONS

Notation and Numeration of Decimals

242. The orders of decimals below thousandths are ten-thousandths, hundred-thousandths, millionths, etc., as shown in the following table.

The orders below millionths are ten-millionths, hundred-millionths, billionths, ten-billionths, etc. They are seldom used.

Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
3	6	5	.	7	4	3	2	9	8
	4	0	.	0	0	2	4		
	8	8	.	3	0	6	9	6	

1. 365.743298 is read "365 *and* 743,298 *millionths.*"
   40.0024 is read "40 *and* 24 *ten-thousandths.*"
   88.30696 is read "88 *and* 30,696 *hundred-thousandths.*"

2. What decimal place is occupied by

Thousandths? Ten-thousandths? Hundred-thousandths?
Millionths? Ten-millionths? Hundred-millionths?

3. How many decimal figures are required to express thousandths? ten-thousandths? hundred-thousandths? millionths? ten-millionths? hundred-millionths?

4. 1, .1, .01, .001, .0001, .00001, .000001.
What part is each decimal of the number on its left?
How many tenths are there in 1? hundredths in .1? etc.

In the Arabic or decimal system of notation, a unit of any order is $\frac{1}{10}$ of the next higher or left-hand unit, and 10 times the next lower or right-hand unit.

In reading a decimal, it should be read as an integer, and the denomination of the right-hand figure should be added.

In reading a mixed decimal the word "and" is used between the integral and decimal parts, and not elsewhere.

EXERCISES

243. Read:

1. .44	**8.** .2563	**15.** 6.7561
2. .044	**9.** .02563	**16.** 4.2837
3. .144	**10.** .00492	**17.** 10.0361
4. .0144	**11.** .06007	**18.** 78.260005
5. .0072	**12.** .36091	**19.** 34.035462
6. .4503	**13.** .548273	**20.** 217.38527
7. .7902	**14.** .048273	**21.** 3654.0728

WRITTEN EXERCISES

244. Express in figures:

1. 2 hundreds and 25 thousandths.
2. 20 units and 733 thousandths.
3. 625 units and 85 ten-thousandths.
4. 16 thousands, 382 units, and 95 millionths.
5. 485 millions, 7 thousands, and 17 thousandths.
6. 1 million, 1 thousand, 1 unit, and 1 hundred-thousandth.
7. Seventy-five and twenty-one thousandths.
8. Ten and three thousand one hundred six millionths.
9. Ninety-six and four hundred ninety ten-thousandths.

10. Six hundred sixty-six thousand six hundred sixty-six millionths.

11. Six hundred sixty-six thousand and six hundred sixty-six millionths.

12. Four hundred seventeen thousand two hundred six millionths.

13. Four hundred seventy-seven thousand and two hundred sixty-nine millionths.

14. Ninety-six and thirty-two ten-thousandths.

15. Two hundred sixty and three hundred fifteen hundred-thousandths.

Write in words:

16. 401	18. .401	20. .0366	22. 2000.002
17. 400.001	19. 401,000	21. 300.0066	23. 2002.002

Reduction of Decimals

245. Reduction of decimals to common fractions.

WRITTEN EXERCISES

1. Reduce .0625 to a common fraction.

SOLUTION. $.0625 = \frac{625}{10000} = \frac{1}{16}$.

Reduce to common fractions in their lowest terms:

2. .0625	9. .03125	16. .53125
3. .1875	10. .09375	17. .65625
4. .3125	11. .15625	18. .78125
5. .4375	12. .21875	19. .96875
6. .5625	13. .28125	20. .890625
7. .6875	14. .34375	21. .609375
8. .8125	15. .40625	22. .484375

246. Reduction of common fractions to decimals.

WRITTEN EXERCISES

1. Reduce $\frac{4}{7}$ to a six-place decimal.

$$7\overline{)4.000000}\quad .571429-$$

$\frac{1}{7}$ of 40 tenths = 5 tenths and 5 tenths remaining; $\frac{1}{7}$ of 5 tenths or of 50 hundredths = 7 hundredths and 1 hundredth remaining; and so on. The last division gives $\frac{1}{7}$ of 60 millionths = $8\frac{4}{7}$ millionths. Since $8\frac{4}{7}$ millionths is nearer 9 millionths than 8 millionths, 9 is written in the quotient rather than 8; but a small minus sign is written after the 9 to show that the true quotient is a little less than .571429.

In the following, no results should be carried beyond six decimal places.

2. Reduce $2\frac{4}{7}$ to a six-place decimal.

By Long Division

$$\begin{array}{r} .148148+ \\ 27\overline{)4.0000} \\ 27 \\ \hline 130 \\ 108 \\ \hline 220 \\ 216 \\ \hline 40, \text{ etc., as from the beginning.} \end{array}$$

By Short Division

$$\begin{array}{r|l} 3 & 4.000000 \\ 3 & 1.333333+ \\ 3 & .444444+ \\ \hline & .148148+ \end{array}$$

The small plus signs show that the true quotients are a little larger than those set down, but less than $\frac{1}{2}$ millionth larger.

After finding the first three figures of the quotient by long division, it is found that the new dividend is like the original dividend. Hence, the next three figures of the quotient will be like the first three; and in fact the same set of figures will recur, however far the division is carried.

The short division process is advised when the divisor can be separated readily into factors.

Reduce to decimals, not beyond six places:

3. $\frac{1}{4}$	10. $\frac{5}{6}$	17. $\frac{5}{12}$	24. $\frac{9}{32}$
4. $\frac{1}{8}$	11. $\frac{5}{8}$	18. $\frac{5}{16}$	25. $\frac{13}{32}$
5. $\frac{3}{8}$	12. $\frac{3}{7}$	19. $\frac{7}{16}$	26. $\frac{25}{32}$
6. $\frac{1}{6}$	13. $\frac{4}{5}$	20. $\frac{9}{16}$	27. $\frac{11}{12}$
7. $\frac{3}{4}$	14. $\frac{5}{9}$	21. $\frac{11}{16}$	28. $\frac{4}{15}$
8. $\frac{7}{8}$	15. $\frac{4}{7}$	22. $\frac{13}{16}$	29. $\frac{9}{64}$
9. $\frac{3}{16}$	16. $\frac{2}{11}$	23. $\frac{5}{32}$	30. $\frac{33}{64}$

Addition and Subtraction of Decimals

WRITTEN EXERCISES

247. The following have been added and tested in 9 minutes. Practice until you can do as well.

1.	2.	3.	4.
98.046	3.8649	3.1416	5.2064
48.792	8.0095	4.8	6.7187
25.4	.8008	2.75	4.6562
3.96	32.84	4.853	3.4687
85.244	1.7854	.9244	9.28125
38.008	86.4139	1.3065	45.05625

5.	6.	7.	8.
11.60625	.03794	.222462	45.6425
5.21875	.03826	.225729	38.8448
.04375	.05023	.227726	40.0505
2.53125	.06199	.248495	68.2734
4.59375	.07368	.270805	9.9999
16.00625	.09644	.299573	18.2184

Subtract:

9. 4.2532 3.8216	10. 8.4605 4.2841	11. 7.5225 3.1895
12. 25.6250 7.1849	13. 88.3300 6.4727	14. 3.42 0.8625
15. 7 6.48156	16. 10 3.141593	17. 5 0.006995

Subtract from 1:

18. .0625	20. .4375	22. .40625	24. .071875
19. .3099	21. .1827	23. .53667	25. .222414

26. Syracuse is situated on a railroad 439.44 miles long between New York and Buffalo, and is 148.72 miles from Buffalo. How far is Syracuse from New York?

27. The German mark is worth $.2385, the French franc $.193, and the English shilling $.2434. How much more is the shilling worth than the franc? than the mark?

How much less than a quarter of a dollar is each of these foreign coins worth?

28. A nail 5 inches long is driven through a board so that it projects 2.419 inches on one side and 1.706 inches on the other. How thick is the board?

29. If a cubic foot of coal weighs 82.9628 lb. and of pine 41.0132 lb., how much more does 1 cu. ft. of coal weigh than 2 cu. ft. of pine?

30. Recently the total mileage of the Canadian Pacific Railway was 8667.5 miles. This was 2973.53 miles shorter than that of the Southern Railway and the Baltimore and Ohio combined, but 1468.82 miles longer than the Southern Railway alone. How long was each road?

31. The fastest time made by ships in crossing the Atlantic Ocean for each decade since 1850 has been: 217.75 hr., 190.05 hr., 178.883 hr., 139.3 hr., and 127.383 hr. Find the time gained each decade.

32. If the average weight of a boy is 61.28 lb. at 10 years of age, 64.89 lb. at 11 years, 72.55 lb. at 12 years, 78.32 lb. at 13 years, 87.41 lb. at 14 years, and 103.29 lb. at 15 years, find the average gain in weight for each year.

33. Some onions were raised on a plot of ground at a cost of $17.44 for growing them, $1.13 for gathering, and $10.95 for trimming and bunching. If they were sold for $131.51, how much profit was made?

Multiplication of Decimals

248. 1. Find the product of:

.2 and 3; .02 and 3; .002 and 3.

What is the product of tenths and units? of hundredths and units? of thousandths and units?

2. Since $.2 = \frac{2}{10}$ and $.3 = \frac{3}{10}$, what is the product of .2 and .3? of .02 and .3? of .002 and .3?

What is the product of tenths and tenths? of hundredths and tenths? of thousandths and tenths?

3. What is the product of hundredths and hundredths? of hundredths and thousandths?

4. Multiply:

.2	.2	.02	.002	.002	.02	.002
3	.3	.3	3	.3	.03	.03

5. How may the number of decimal places in the product be obtained from the number of decimal places in the multiplicand and multiplier?

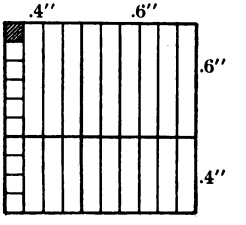
EXERCISES

249. Answer quickly:

1. .2 of 3	**5.** $4 \times .7$	**9.** $.05 \times .11$
2. .2 of .3	**6.** $.4 \times .7$	**10.** $.5 \times .025$
3. .2 of .03	**7.** $.04 \times .9$	**11.** $.015 \times .03$
4. .2 of .003	**8.** $.004 \times .3$	**12.** $.13 \times .0002$

13. What part of a square inch is .1 of .1 of it? .4 of .1 of it? .6 of .1 of it? .9 of .1 of it?

14. How many hundredths of a square inch are there in a square, each side of which is .4 in. long? in a rectangle .4 in. by .6 in.?



15. Find the area of a rectangle .6 in. by .4 in.; .6 in. by .6 in.

16. Find the area of a rectangle 1.2 in. by .5 in.; 7 ft. by .3 ft.; 1.5 ft. by .4 ft.; 1.1 rd. by .9 rd.

250. *The number of decimal places in the product is equal to the number of decimal places in both multiplicand and multiplier.*

WRITTEN EXERCISES

251. **1.** Multiply .63 by .48.

$$\begin{array}{r} .63 \\ .48 \\ \hline 504 \\ 252 \\ \hline .3024 \end{array}$$

The product of 63 and 48 is 3024.

Since the multiplicand .63 has two decimal places and the multiplier .48 two decimal places, the product must contain $2 + 2$, or *four*, decimal places.

Therefore, the product is .3024.

Multiply:

2. $.49 \times .56$	**6.** $.123 \times .345$
3. $.84 \times .36$	**7.** $.871 \times .413$
4. $.971 \times .78$	**8.** $.649 \times .507$
5. $.842 \times .36$	**9.** $.987 \times .403$

10.	11.	12.
.32	.34	60.29
.26	.45	.095
192	170	30145
64	136	54261
.0832	.153ø	5.72755

Decimal ciphers are *prefixed*, when necessary, to give the product the proper number of decimal places.

Decimal ciphers on the right of the product are *omitted*.

Multiply :

13. .71 × .84
14. .49 × .28
15. .46 × 3.54
16. .67 × 10.22
17. .421 × 8.45
18. .545 × 3.05
19. .625 × 2.64
20. .875 × 8.56
21. 2.165 × 4.306
22. 5.008 × 7.24
23. 9.243 × 10.07
24. 1.008 × .875
25. 2.041 × .076
26. 1.001 × .008
27. 3.564 × 18.245
28. 8.069 × 100.004

29. If the railroad fare between Chicago and San Francisco is $62.50, and the fare in 1880 was 1.856 times as much, what was the fare then?

30. There are 30.8952 miles of asphalt pavement in a certain city. If the total length of paved streets is 2.75 times as much, how many miles of paved streets are there?

31. If it costs $5.94 to irrigate one acre of ground in Colorado, how much does it cost to irrigate 87.5 acres?

32. A California farmer raises 27.5 acres of sugar beets that average 9.81 tons to the acre. If he sells his beets for $4.40 a ton, how much money does he receive?

33. Find the cost at $.12 per bushel of enough cotton seed to plant 245 acres of land, if each acre requires 1.75 bushels.

Division of Decimals

252. Divide:

1. 2)8 2 tenths)8 tenths .2).8 .02).08 .002).008

2. Compare the quotient obtained by dividing .8 by .2 with that obtained by dividing 8 by 2.

Compare .02).08 with 2)8.

Compare .002).008 with 2)8.

3. If the divisor is multiplied by 10, by what must the dividend be multiplied? if the divisor is multiplied by 100? by 1000?

Multiplying both dividend and divisor by the same number does not change the quotient.

4. When the divisor is a decimal, we make the division easier by multiplying both dividend and divisor by a number that will change the *divisor* to an *integer*.

5. Change .2).84 to 2)8.4 and then divide.

6. Change .02).184 to 2)18.4 and then divide.

7. How should the dividend and the divisor be prepared for division when the divisor contains one decimal figure? two decimal figures? any number of decimal figures?

253. *The divisor may be changed to an integer, without changing the value of the quotient, by moving the decimal points in both dividend and divisor toward the right as many places as there are decimal figures in the divisor.*

When the dividend and divisor have been prepared for division by changing the divisor to an integer, the decimal point of the quotient should be placed vertically above or below that of the dividend.

EXERCISES

254. Divide:

1. .3)$\overline{.6}$	**7.** 1.2)$\overline{8.4}$	**13.** .004)$\overline{.024}$
2. .4)$\overline{.8}$	**8.** 1.1)$\overline{1.32}$	**14.** .003)$\overline{1.236}$
3. .2)$\overline{1.2}$	**9.** .09)$\overline{.729}$	**15.** .008)$\overline{1.688}$
4. .3)$\overline{1.5}$	**10.** .07)$\overline{.042}$	**16.** .012)$\overline{1.440}$
5. .6)$\overline{4.2}$	**11.** .11)$\overline{.099}$	**17.** .011)$\overline{.0099}$
6. .8)$\overline{9.6}$	**12.** .08)$\overline{.848}$	**18.** .012)$\overline{.000096}$

19. Divide by .5: .25, .125, .0625, 20.5.

20. If 1.2 inches of a candle burn in one hour, in how many hours will 7.2 inches burn?

21. If a railroad train travels .9 of a mile in one minute, how many minutes does it take to travel 10.8 miles?

22. If Henry expends \$.05 a day for car fare, how long will \$1.50 last him?

23. If one pound of grapes makes .3 of a pound of raisins, how many pounds of grapes are required to make 10.5 pounds of raisins?

24. A blacksmith bought an anvil for \$9, paying \$.10 per pound for it. How much did the anvil weigh?

WRITTEN EXERCISES

255. **1.** Divide .1296 by .048.

.1296 ÷ .048

$$\begin{array}{r} 2.7 \\ 48\overline{)129.6} \\ 96 \\ \hline 336 \\ 336 \\ \hline \end{array}$$

Since the divisor expresses thousandths, to change the divisor to an integer *without changing the quotient*, both dividend and divisor are multiplied by 1000 by moving the decimal point in each three places toward the right.

Divide:

2. 8.75 by 3.5
3. 15.18 by 4.6
4. 31.68 by 4.4
5. 50.40 by 7.5
6. 57.60 by 12.8
7. 54.78 by 6.6
8. 3.268 by .76
9. 3.612 by .43
10. 10.962 by 8.7
11. 4.872 by 1.45
12. 8.906 by 3.65
13. 42.602 by 8.95
14. 48.300 by 6.44
15. 10.897 by 4.25

16. $22.496 \div 3.2$

$$\begin{array}{r} 7.03 \\ 32\overline{)224.96} \\ \underline{224} \\ 96 \\ \underline{96} \end{array}$$

17. $.011 \div 4.4$

$$\begin{array}{r} .0025 \\ 44\overline{).1100} \\ \underline{88} \\ 220 \\ \underline{220} \end{array}$$

18. $5 \div .15625$

$$\begin{array}{r} 32 \\ 15625\overline{)500000} \\ \underline{46875} \\ 31250 \\ \underline{31250} \end{array}$$

When the dividend contains fewer decimal figures than the divisor, the deficiency should be made up, before moving the decimal point, by annexing decimal ciphers. Thus, exercise 18, $5 \div .15625 = 5.00000 \div .15625$.

19. .021 by 2.8
20. .0198 by 3.6
21. .0162 by 4.5
22. .0403 by .65
23. .6345 by .75
24. .0546 by .65
25. .02275 by .625
26. .03262 by 93.2
27. .02346 by .68
28. 5.000 by .625
29. 1365 by .8125
30. 15 by .46875
31. 29 by .453125
32. 161 by .71875
33. 15.3 by .53125
34. 794.64 by 32.25
35. 91.6243 by 18.38
36. 66.112 by .01033
37. .009604 by .0056
38. 2352.9122 by 36.664
39. $47,787.50 by $76.46
40. $17,253.55 by $42.34

256. 1. Find the value of $.4 \times .6$; of $\frac{.4 \times .6}{.12}$.

2. How will the value of the dividend be affected if *one* of the factors, as .4, is multiplied by 10? if *both* factors are multiplied by 10?

3. If both factors of the dividend are multiplied by 10, by what number must the divisor be multiplied to prevent the quotient from being changed?

4. How, then, does $\frac{.4 \times .6}{.12}$ compare in value with $\frac{4 \times 6}{12}$?

WRITTEN EXERCISES

257. 1. Divide $7.5 \times 1.8 \times .33$ by $9.9 \times .25 \times 12$.

$$\frac{\cancel{7.5} \times \cancel{1.8} \times \cancel{.33}}{\cancel{9.9} \times \cancel{.25} \times \cancel{12} \times 10} = \frac{3}{20} = .15$$

Omitting decimal points from the dividend multiplies it by $10 \times 10 \times 100$, or by 10,000. Omitting decimal points from the divisor multiplies it by 10×100, or by 1000. Consequently, to free both dividend and divisor of decimals *without changing the quotient*, after omitting decimal points we must multiply the divisor by 10. By cancellation the quotient is then found to be $\frac{3}{20}$, or .15.

Divide:

2. 4.2×1.6 by .56
3. 9.5×1.3 by .38
4. $.44 \times 2.1$ by 1.32
5. 4.8×2.8 by .032
6. 2.88 by $.8 \times 2.4$
7. .168 by $.35 \times 1.2$
8. 8.1 by 3.6×1.8
9. 4.5 by 7.2×1.25
10. $4.2 \times 2.5 \times 7.2$ by $3.5 \times .75 \times 24$
11. $1.5 \times .48 \times 1.05$ by $.45 \times 2.5 \times .16$
12. $.216 \times 7.5 \times 1.68$ by $.315 \times .48 \times .96$
13. $5.5 \times 1.12 \times 7.8 \times .54$ by $.024 \times 2.6 \times .99$
14. $.9 \times .022 \times 2.5 \times 25.2$ by $.63 \times 37.5 \times 4.8$
15. $11.52 \times .105 \times .1728$ by $.048 \times 22.4 \times 7.5 \times .072$

WRITTEN EXERCISES

258. 1. If a farmer sells potatoes for $.65 per bushel and receives $1145.30 for them, how many bushels does he sell?

2. A planter in Alabama pays $519.65 per year for the rent of his farm, at $4.75 per acre. How many acres does he hire?

3. A man paid $27.50 per month for the rent of a house, and after paying $467.50 moved out. How many months did he occupy the house?

4. A chest of tea was sold for $28.31. If the price per pound was $.38, how many pounds did the chest contain?

5. How many pounds are there in a sack of coffee that sells at $.135 a pound and brings $18.90?

6. At $2.85 a box, how many boxes of oranges does a wholesale dealer buy for $2485.20?

7. When 4.8 bushels of wheat make a barrel of flour, how many barrels will 648 bushels of wheat make?

8. A miller in Minnesota sold one day's product of flour for $8002.80, at $3.42 per barrel. How many barrels did the mill turn out that day?

9. If the ruble of Russia is worth $.515 in our money, how many rubles are there in $638.60?

10. The area of a field is 350.2 square rods, and one side is 13.6 rods long. Find the length of the other side.

11. A wall map is 24.3 inches long, and has an area of 675.54 square inches. Find its width.

12. The cost of drilling a well at $.875 per foot was $1333.50. How deep was the well?

13. A train went from Kendallville, Ind., to Toledo, Ohio, a distance of 91 miles, in 1.25 hours. At what rate per hour did the train run?

MISCELLANEOUS EXERCISES

259. 1. The strawberries grown on 4.5 acres of land were sold for $2377.35. What was the income per acre?

2. A cow gave 5738.35 lb. of milk in a year. How many quarts of milk did she give, if 1 quart weighs 2.15 lb.?

3. The cost of building a new road 21.7 miles long was $6983.06. What was the cost per mile?

4. A California road 25 miles long was sprinkled with petroleum, 175.5 barrels to the mile. Find the cost of the petroleum at 70¢ per barrel.

5. A farmer drew his produce to market, a distance of 6.75 miles, at a cost of $1.755 per load. What was the cost of drawing a load per mile?

6. If it requires a flow of 3.77 gallons of water per minute to irrigate 1 acre of land, how many acres can be irrigated by a flow of 207.35 gallons per minute?

7. If .92 of an iceberg is under water, how many cubic feet of ice are there in an iceberg that has 196,880 cubic feet beneath the surface?

8. In the United States the average cost of living in 1860 was $115.191 per person, and in 1904, $97.192. How much less on the average did a year's living for a family of 5 persons cost in 1904 than in 1860?

9. It is 89.885 miles from Albion to Utopia, and 101.215 miles from Utopia to Carthage. Find the cost, at $.13 per mile, of the coal consumed by a locomotive in making a run from Albion to Carthage.

10. One year an electric company operated 128.18 miles of its own track and 24.28 miles of track leased from another company. The operating expenses of the road were $334,421.01. Find the operating expenses per mile of track.

REVIEW PROBLEMS IN INDUSTRIES

260. **1.** A fair-sized colony of bees averages 1 queen, 750 drones, and about 2700 workers. How many bees are there in such a colony?

2. A man in Missouri bought 140 colonies of bees at \$4.95 per colony. How much did he invest in bees?

3. He bought also 18,000 honey boxes at \$3 per thousand, 280 supers (to be placed on the hives for receiving the honey boxes) at 25¢ each, 780 shipping cases at 16¢ each, and 100 pounds of comb foundation at 52¢ per pound. Find the cost of these supplies.

4. Find the cost of labor for 5 months: 1 boy at \$10 per month, 1 girl at \$10 per month, 1 woman at \$30 per month; his own labor being worth \$50 per month.

5. His entire output was 15,520 pounds of honey; $\frac{5}{8}$ of it was clover honey and the rest buckwheat honey. How many pounds of each kind were there?

6. The average price received for the clover honey (9700 lb.) was $12\frac{1}{2}$¢ per pound and for the buckwheat honey (5820 lb.) $10\frac{1}{2}$¢ per pound. How much did he receive for the honey?

7. One year 125,000,000 pounds of honey were produced in the United States. Find its value at 9¢ per pound.

One year two boys and two girls rented a lot next to their house for a garden. Part of the lot had been used as a garden before, and contained some berry bushes.

Some of the vegetables paid only for the cost of raising, but the following proved to be profitable:

Crop	Seed	Yield
Beets	1 ounce @ 10¢	24 bunches
Peas	2 pints @ 12½¢	5½ pecks
Tomatoes	2 packets @ 5¢	6¾ bushels
Lima beans	1½ pints @ 10¢	1½ bushels
Currants	.....	43 quarts
Gooseberries	.....	21 quarts
String beans	2 pints @ 15¢	2 bushels 1 peck
Lettuce	3 packets @ 5¢	160 heads
Sweet corn	2½ pints @ 12¢	330 ears

8. How much did the seed cost?

Find the proceeds from:

9. Beets @ 5¢ per bunch.

10. Peas @ 40¢ per pk.

11. Tomatoes @ 20¢ per pk.

12. Lima beans @ 8¢ per qt.

13. Currants @ 11¢ per qt.

14. Gooseberries @ 12¢ per qt.

15. Find the sum received for lettuce, if $\frac{1}{2}$ of the crop sold at 5¢ per head, $\frac{3}{8}$ of the crop at 6¢ per head, and the rest of it at 2 heads for 15¢.

16. The string beans were grown by the children in two lots,

the first of which was $\frac{1}{3}$ of the whole yield, and brought 15¢ per quart; the second, picked in September, sold at 10¢ per quart. How much was received for string beans?

17. There were two crops of corn, one early and one late. The early crop yielded 132 ears, which sold at 20¢ per dozen, while the late crop sold at 16¢ per dozen. How much did the corn bring?

18. The tomato seed they had planted in a box in late winter, and had thus raised 90 young plants, 24 of which they set out for themselves. They sold the rest at 30¢ per dozen. How much did their tomato plants bring?

19. What were the total receipts for vegetables and plants?

20. One corner of the garden was devoted to raising flowers. A packet of aster seed produced 40 healthy plants which yielded 6 fine flowers apiece. At 25¢ per dozen flowers, how much money did the asters bring?

21. The first sweet peas were picked July 14, the last October 6. If the average number of stems picked per day was 36, how many were obtained during the season?

22. These flowers were sold at 18¢ per bunch of 36 stems. Find the proceeds from sweet peas.

23. The girls also planted some choice dahlia bulbs, and from the mature plants secured that summer 672 dahlias. Find the sum obtained by selling the dahlias at 20¢ per dozen.

24. What were the total receipts from flowers? from the whole garden?

25. The expenses of the garden were: cost of vegetable seed as found in exercise 8; 4 packets of flower seed @ 10¢; 8 dahlia bulbs @ $2.85 per dozen; $4.29 for fertilizer; $8.50 for rent. Find the total cost.

26. Find the total profits from the garden, and each child's share, if the profits were divided equally.

Elinor found this clipping in a newspaper:

> Twelve million six hundred thousand was the number of famous Rocky Ford cantaloupes shipped from the Rocky Ford district in Colorado, this season. 700 car loads were sent out, as against 592 car loads the previous year.

27. How many more car loads were shipped this year than the previous year?

28. What was the average number of cantaloupes shipped per car this year?

29. How many crates were shipped, if the average number of cantaloupes in a crate was 45?

30. How many such crates were there in an average car load of 18,000 cantaloupes?

31. A crate will hold 30 large cantaloupes, 45 of standard size, or 64 small ones. How many cantaloupes are there in a car containing 150 crates of each size?

32. Compare the number of cantaloupes (20,850) in such a car load with the number in a car load of 450 standard crates.

33. A grower's first large picking of cantaloupes occurred August 9, or 100 days after he had planted the seed. When did he plant the seed?

34. At this picking he obtained 5000 cantaloupes, of which .8 were standard size, .07 large size, and the rest small size. How many were there of each size?

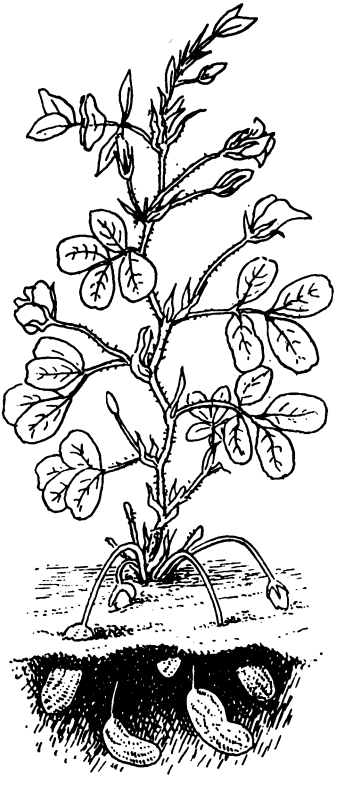
35. A wholesale dealer in New York bought a car load of South Carolina cantaloupes containing 467 crates @ \$1.10. Freight and icing charges were \$226.50. He sold 146 crates @ \$2.25, 181 @ \$2.10, and 140 @ \$2.45. Find his gain.

36. A retailer bought a crate of 45 cantaloupes for \$2.50. He sold 23 @ 10¢, 18 @ 8¢, and the rest @ 6¢. Find his gain.

37. The steward of a club bought a crate of 30 choice cantaloupes for \$5. They were served at the table for 30¢ per half cantaloupe. What was the profit on the crate?

38. A farmer in Tennessee had a piece of ground 52 rods by 20 rods, devoted to peanut raising. How many acres did it contain?

For the $6\frac{1}{2}$ acres, find the cost of:



39. Seed, 2 bushels per acre, at \$1.02 per bushel.

40. Preparing the ground, \$1.90 per acre.

41. Planting, \$1.10 per acre.

42. Cultivating and caring for the crop, \$4.72 per acre.

43. Harvesting the crop, \$16.50 per acre.

44. What was the total expense and what was the expense per acre?

45. The field produced $354\frac{1}{4}$ bushels of peanuts. What was the average yield per acre?

46. How much was received for the crop, at 96¢ per bushel?

47. Find the profit on the crop.

48. In addition to the cost, \$340.08, the buyer paid \$50.70 to ship the peanuts to Chicago. He then sold them at \$1.24 per bushel. Find his gain.

49. The buyer in Chicago sold 215 bushels @ \$1.44 and the rest @ \$1.36. How much did he gain, the cost being \$439.27?

50. One year Virginia produced 3,713,347 bu. of peanuts; N.C. 3,460,439 bu.; Ga. 1,435,775 bu.; Ala. 1,021,708 bu.; Fla. 967,927 bu.; and Tenn. 747,668 bu. How many more bushels did Virginia produce than each of the other states?

A gardener in Michigan set out $\frac{3}{4}$ of an acre of strawberries.

EXPENSE OF CULTIVATION

Preparing the ground	$ 2 25
Plants, 7500 at $2 per thousand	
Setting the plants	2 75
Hoeing, etc.	8 50
Weeding, etc.	18 75
Straw, $3\frac{1}{2}$ tons at $4 per ton	
Spreading the straw	1 50
Fertilizer	15 00

EXPENSE OF HARVESTING AND MARKETING

150 bushel crates at $15 per hundred	
4800 quart baskets at $3 per thousand	
Picking 4800 quarts @ $1\frac{1}{2}$ ¢	
Shipping and selling, 54 ¢ per crate	

51. Find the cost of the plants; of the straw; of cultivation.

52. How much did the crates cost? the baskets?

53. Compute the cost of picking; of shipping and selling.

54. Find the expense of harvesting and marketing.

55. What was the gardener's total expense?

RECEIPTS

2 bushel crates @ $5.12	
12 " " @ $4.16	
36 " " @ $3.80	
76 " " @ $2.48	
24 " " @ $1.94	

56. Compute the gardener's receipts for his crop.

57. How much did he gain?

58. Find the average price received for the crop per crate; per quart; the highest price per quart; the lowest.

59. What was the yield in quarts per acre?

PART II

PRELIMINARY REVIEW

261. 1. The following table shows the number of days' attendance in the graded schools of a city.

Find the total attendance for each month; then find the total attendance in each school; then find the total attendance in all the schools during the year, by adding the totals for each month, and also by adding the totals for each school.

School	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Totals for each school
Lincoln	4824	5236	5361	4550	5555	4482	5310	4965	4946	4461	
Greeley	9042	9864	9430	8631	9998	8535	9787	8960	9362	7751	
Park	3977	4385	4202	3872	4724	3636	4427	4016	4215	3366	
Douglass	5055	5432	5225	4833	5752	4500	5499	5005	5176	4353	
Irving	5987	6548	6367	5749	6900	5484	6518	6000	6283	5244	
White	8101	8736	8452	7686	9275	7310	8797	8050	8436	6787	
Whittier	5166	5490	5273	4900	5804	4516	5500	5096	5230	4375	
Longfellow	8104	8821	8330	7651	9352	7553	8890	8172	8472	6785	
Totals for each month											

2. Find the average monthly attendance for each school. How much greater was it for the Greeley School than for the Longfellow School?

3. Find the average attendance at all the schools for each month. How much greater was it for January than for December?

4. Find the average daily attendance for all the city schools, regarding 200 days as a school year.

Add and test in 3 minutes or less:

5.	6.	7.	8.
428.356	362.44	428.114	504.405
29.438	171.86	32.56	367.242
51.097	93.55	9.29	596.385
34.253	47.286	8.47	89.755
8.756	9.099	82.625	7.4836
11.482	12.764	75.075	5.2948
15.943	346.201	54.343	8.3006
506.042	798.45	281.983	11.5998

Subtract rapidly:

9. 34.645	10. 52.000	11. 100.000	12. 1000.00
5.256	7.248	5.272	524.38
13. 75.05	14. 3.864	15. 10	16. 25
3.4642	.2457	1.4265	6.4873

Multiply:

17. 650 by 4.2
18. 3.87 by .4
19. 5.65 by .24
20. 3.87 by 6.2
21. .875 by 8.09
22. .4964 by .55.
23. .0284 by 47.5
24. 16.288 by 5.305
25. 19.082 by 156.7
26. 586.48 by .0105

Factor the following numbers:

27. 102	31. 248	35. 1452	39. 85,050
28. 201	32. 729	36. 1296	40. 15,876
29. 405	33. 1728	37. 1050	41. 20,736
30. 504	34. 2160	38. 3650	42. 41,088

43. What is an even number? an odd number?
Which of the numbers in exercises 27–42 are even? odd?

Reduce to lowest terms:

44. $\frac{45}{60}$ 47. $\frac{80}{96}$ 50. $\frac{15}{75}$ 53. $\frac{95}{380}$

45. $\frac{26}{39}$ 48. $\frac{75}{125}$ 51. $\frac{49}{56}$ 54. $\frac{875}{1200}$

46. $\frac{39}{65}$ 49. $\frac{44}{110}$ 52. $\frac{48}{112}$ 55. $\frac{1382}{5280}$

Find the sum and the difference of:

56. $\frac{3}{8}$ and $\frac{5}{16}$ 60. $\frac{4}{3}$ and $\frac{7}{8}$ 64. $3\frac{5}{8}$ and $2\frac{3}{4}$

57. $\frac{7}{8}$ and $\frac{11}{16}$ 61. $\frac{15}{16}$ and $\frac{9}{10}$ 65. $9\frac{2}{3}$ and $5\frac{5}{7}$

58. $\frac{3}{4}$ and $\frac{2}{3}$ 62. $1\frac{5}{6}$ and $1\frac{1}{12}$ 66. $7\frac{16}{25}$ and $4\frac{17}{20}$

59. $\frac{2}{3}$ and $\frac{8}{9}$ 63. $2\frac{1}{2}$ and $1\frac{7}{12}$ 67. $4\frac{38}{75}$ and $3\frac{7}{50}$

68. What may be done to the terms of a fraction without changing its value?

69. What are similar fractions?

70. What must be done to fractions that are not similar before they can be added or subtracted?

Perform the operations indicated:

71. $2\frac{1}{4}+3\frac{5}{8}-1\frac{13}{16}$ 74. $4\frac{3}{4}-1\frac{2}{3}+\frac{9}{15}+2\frac{2}{5}$

72. $5\frac{1}{2}-2\frac{7}{10}+15\frac{4}{5}$ 75. $6\frac{1}{2}-4\frac{1}{3}+8\frac{3}{4}-9\frac{5}{6}+\frac{2}{5}$

73. $9\frac{1}{4}-2\frac{5}{6}-3\frac{2}{3}$ 76. $20-1\frac{1}{6}-2\frac{3}{5}-4\frac{7}{8}-9\frac{3}{4}$

Reduce to improper fractions:

77. $3\frac{9}{10}$ 79. $4\frac{11}{12}$ 81. $33\frac{1}{3}$ 83. $87\frac{1}{2}$

78. $6\frac{7}{8}$ 80. $15\frac{4}{25}$ 82. $66\frac{2}{3}$ 84. $83\frac{1}{3}$

Multiply or divide as indicated:

85. $1\frac{5}{7}\times 4\frac{3}{8}$ 90. $16\frac{4}{5}\times 12\frac{1}{2}$ 95. $266\frac{2}{3}\div 33\frac{1}{3}$

86. $5\frac{1}{15}\times 21\frac{3}{16}$ 91. $56\frac{7}{8}\div\frac{7}{12}$ 96. $500\div 16\frac{2}{3}$

87. $3\frac{6}{25}\times 27\frac{}{9}$ 92. $18\frac{2}{3}\div 4\frac{28}{33}$ 97. $350\div 87\frac{1}{2}$

88. $4\frac{2}{3}\div 3\frac{1}{21}$ 93. $14\frac{2}{5}\times 10\frac{5}{12}$ 98. $625\div 37\frac{1}{2}$

89. $14\frac{7}{8}\div 3\frac{1}{2}$ 94. $90\div 23\frac{7}{16}$ 99. $1000\div 62\frac{1}{2}$

100. Tell why $\frac{1}{2}\div\frac{3}{4}=\frac{1}{2}\times\frac{4}{3}$. Why is the divisor inverted?

101. Compare $\frac{7}{8}$ with $\frac{7}{10}$ in two ways, as follows:

(*a*) Find how much greater or less $\frac{7}{8}$ is than $\frac{7}{10}$.

(*b*) Find how many times $\frac{7}{8}$ contains $\frac{7}{10}$.

102. Compare $\frac{7}{12}$ with $\frac{7}{10}$ in two ways.

103. The standard size adopted for common brick is $8\frac{1}{4}$ in. by 4 in. by $2\frac{1}{4}$ in. Find the volume of a brick of standard size.

104. How much greater or less in volume is each of the following than a brick of the standard size:

Milwaukee brick, $8\frac{1}{2}$ in. by $4\frac{1}{8}$ in. by $2\frac{3}{8}$ in.?
Maine brick, $7\frac{1}{2}$ in. by $3\frac{3}{8}$ in. by $2\frac{3}{8}$ in.?
North River brick, 8 in. by $3\frac{1}{4}$ in. by $2\frac{1}{4}$ in.?

105. To how many standard bricks are 1000 Milwaukee bricks equivalent in volume? 1000 North River bricks?

Multiply:

106. .77 by .24

107. .38 by 5.6

108. .72 by .025

109. 9.6 by .288

110. .0125 by 4.4

111. .0065 by .72.

112. .0327 by .09

113. 1.625 by 7.6

114. 3840 by .875

115. 43.11 by $.66\frac{2}{3}$

116. 105.602 by $.12\frac{1}{2}$

117. 360.044 by $.37\frac{1}{2}$

Divide:

118. .86 by 17.2

119. .7755 by .75

120. .2292 by .048

121. 219.45 by 8.75

122. .15795 by 3.25

123. .02163 by 3.09

124. .485322 by .94

125. 15.54378 by 4.2

126. 4.59459 by .7854

127. 25.76112 by 3.1416

Reduce to a common fraction in its lowest terms:

128. .3125 **129.** .00875 **130.** .01025 **131.** .000225

Reduce to a decimal to the nearest thousandth:

132. $\frac{4}{9}$ **133.** $\frac{7}{9}$ **134.** $\frac{11}{12}$ **135.** $\frac{29}{17}$ **136.** $\frac{527}{288}$

262. 1. If a flour mill turns out 30 barrels of flour per hour and runs continuously from 7 A.M. on Monday to 12 M. on Saturday, how many barrels will it turn out in a year of 52 weeks?

2. A man's gas meter registered 28,470 cubic feet on Jan. 1 and 35,670 cubic feet on Mar. 1. Find the cost of his gas for the quarter, at 90¢ per 1000 cubic feet.

3. Find the freight charges on a car load of steel shipped from Pittsburg to Montreal, weight 48,200 pounds, at $20\frac{1}{2}$¢ per 100 pounds.

4. The average weight of a newspaper having a daily circulation of 240,000, half of which are sent by mail, is $2\frac{7}{8}$ ounces. Find the cost of postage, at 1¢ per pound, for a day; for a year of 313 week days.

5. The "Twentieth Century Limited" made a run of 133.4 miles between Toledo, Ohio, and Elkhart, Ind., in 1 hr. 54 min. How many miles per hour did the train run?

6. On one occasion a short run of 7.29 miles was made by an American train in 4 minutes. At what rate per hour did the train run?

7. A tank 6 feet long, $2\frac{1}{2}$ feet wide, and $2\frac{2}{3}$ feet deep (inside dimensions) has its sides, ends, and bottom lined with sheet lead weighing $3\frac{1}{2}$ pounds per square foot. Find the cost of the lead at 6¢ per pound.

8. Find the cost of draining a meadow by laying 3960 feet of tile at $10 per thousand feet and digging and filling 3960 feet of ditch at $33\frac{1}{3}$¢ per rod.

9. The meadow contained $62\frac{1}{2}$ acres. Before it was drained it was worth $2125, and after it was drained it was valued at $3900. How much per acre was its value increased by drainage?

10. The crew of a passenger train were paid as follows :

Engineer, 3½ ¢ per mile.	2 brakemen, each $60 per mo.
Fireman, 2 ¢ per mile.	Baggage master, $84 per mo.
Conductor, $125 per mo.	3 porters, each $30 per mo.

The daily run was 286 miles. Find the amount of wages paid the crew per month of 30 days.

11. Of three cargoes of bananas unloading at Pier No. 1, the first contained 22,360 bunches from the British West Indies, the second 19,250 bunches from Cuba, and the third 20,780 bunches from Santo Domingo. How many bunches were there in the three cargoes ?

12. A fruit auctioneer sold 26 car loads of oranges, 360 boxes to the car, in 2 hours 10 minutes. How many boxes did he sell per minute ?

13. The auctioneer's receipts from this shipment of oranges were $21,060. How much, on the average, did he receive for one box ?

14. The same auctioneer once sold 21,000 crates of pineapples, or 630,000 pineapples, in 1½ hours, realizing $45,000. Find the average number of pineapples in a crate, the average price received for a crate, and the average number of crates sold per minute.

15. Find the cost of 150 barrels of limes at $4.75 per barrel.

16. If a refrigerator box contains 80 quarts of Florida strawberries, how many quarts of strawberries are there in a car load containing 125 such refrigerator boxes ?

17. If it requires 200 pounds of ice to keep the strawberries in each refrigerator box cool during shipment, how many pounds of ice are required for the whole car load ?

18. Find the cost of the ice at $2.15 per ton.

19. Find the amount received for the 10,000 quarts of strawberries in the Northern market, at 35¢ per quart.

20. When California cherries are packed, 10 pounds are put into a box. One year a California orchard produced 2201 boxes of cherries that were sold at $6\frac{1}{2}¢$ per pound, 6365 boxes that were sold at $5\frac{1}{2}¢$ per pound, and 32,219 pounds sold to a cannery at $7¢$ per pound. Find the value of the whole crop.

21. A fruit grower shipped 1600 ten-pound boxes of cherries to New York, where they were sold at auction at $90¢$ per box. Picking and packing cost $1¢$ per pound, freight and refrigeration $22¢$ per box; and other expenses amounted to $25. Find the amount received less expenses.

22. He had been offered $5\frac{3}{8}¢$ per pound on the trees. Did he gain or lose by shipping the cherries, and how much, if the sum received for them less expenses, was $903?

23. The freight and refrigeration charges on 640 boxes of cherries shipped from San Francisco to Chicago amounted to $121.60. Find the cost of shipping per box.

24. Find the freight charges on a car load of oranges weighing 44,000 pounds, shipped from Los Angeles to Boston at $1.25 per 100 pounds.

25. Find the value of 4200 crates of No. 30 pineapples (that is, 30 in a crate), at $1.25 per crate in Jacksonville; at $2.10 per crate, wholesale, in New York; at $15¢$ apiece retail.

26. When a box of No. 172 (that is, 172 in a crate) oranges sells for $4.30, what is the equivalent price per dozen?

27. I have 240 fowls for which I am offered $12\frac{1}{2}¢$ per pound, or $4.75 per dozen. If the average weight is $3\frac{1}{4}$ pounds, which is the better offer, and how much?

28. The engine for a sugar mill weighed 61,460 pounds, the sugar mill 64,750 pounds, the connecting machinery 28,800 pounds, the boiler 19,200 pounds, and various fittings 6750 pounds. The whole was shipped to me, and I was charged $22\frac{1}{2}¢$ per 100 pounds freight. How much did the freight cost me?

29. On a sugar plantation it was found that the cost of growing the cane and manufacturing and refining the sugar was $82.50 per acre. Find the cost of producing one pound, if the average yield per acre was $1\frac{1}{2}$ tons.

30. Find the gain per acre when the sugar sold for $4.50 per hundredweight.

31. When sirup instead of sugar was manufactured from the cane, the cost of production was $14\frac{4}{5}$¢ per gallon, and an average acre yielded 500 gallons of sirup. How much more per acre did it cost to manufacture sugar at $82.50 than sirup?

32. When sugar sold at $4.50 per hundredweight and sirup at $24\frac{1}{2}$¢ per gallon, how much more or less per acre would have been gained by manufacturing sugar than by manufacturing sirup, their respective costs per acre being $82.50 and $74?

---

33. The Great Northern and City tunnel in London is $3\frac{3}{4}$ miles long. Trains take $13\frac{1}{2}$ minutes to go from one end to the other. What is the average time required to run one mile? What is the average speed per minute?

34. Find the weight of an 8-gallon can full of milk, if the can weighs 15 pounds and the milk 2.15 pounds per quart.

35. If a gallon of water weighs $8\frac{1}{3}$ pounds, how much more will an 8-gallon can of milk weigh than the same can filled with water?

36. A boy bought a piece of woven wire netting 22 feet long and 4 feet high for his sweet peas. How much did it cost at $\frac{3}{4}$¢ per square foot?

37. If a cotton mill consumes 15 bales of cotton, averaging 490 pounds each, every day for 300 days in a year, and if the average yield of cotton per acre is 250 pounds, how many acres of cotton are required to supply the mill for a year?

38. If it costs $3\frac{1}{2}$ ¢ a rivet to drive rivets by ordinary hammers and only $1\frac{1}{4}$ ¢ by machinery, how much will be saved by using the latter method in building a steamboat whose frame contains 200,000 rivets?

39. A car load of wheat weighing 45,000 pounds was unloaded by steam shovels in 18 minutes. A bushel of wheat weighs 60 pounds. How many bushels were unloaded per minute?

---

40. A telephone line was constructed between two villages that were $5\frac{1}{2}$ miles apart. Find the cost of surveying the line at 96 ¢ per mile.

41. The poles were set 132 feet apart. How many poles were required? Find their cost at $1.25 each.

42. It cost 19 ¢ to dig the hole for each pole. What was the cost of digging the 220 holes?

43. Every pole was fitted with 2 oak brackets, each mounted with a pony glass insulator and fastened with 2 spikes. Find the total cost of fitting the poles at 5 ¢ apiece for labor, $12 per thousand for brackets, $21 per thousand for insulators, and $\frac{3}{8}$ ¢ apiece for spikes.

44. What was the cost of setting the 220 poles at 17 ¢ each?

45. Two galvanized iron wires, each weighing 165 pounds per mile, were strung the entire distance of $5\frac{1}{2}$ miles. Find, to the nearest cent, the cost of the wire at $4\frac{3}{4}$ ¢ per pound.

46. How much did it cost for trimming trees and stringing the wires at $6 per mile?

47. Find the total cost of constructing the line.

48. The population of the village at one end of the line was 6517, and at the other 6731. Soon after the completion of the line, 1 person out of every 32 was a subscriber. How many subscribers were there in both villages?

49. At $14.25 for each telephone, how much did the company expend for the 414 instruments?

50. Find the annual income from subscribers, if each paid $2.50 per month.

51. In one year the total number of calls by subscribers was 776,250, costing $12,420. What was the average cost of each call?

52. The manager of a city telephone company made 100 calls on the telephone to find how long it took the operators at the exchange to make connections for customers. The connections for the 100 calls were made in 4 min. 7 sec. Find the average time of making one connection.

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53. At an experiment station in New York, 39 ducks and 39 chickens were fed for a period of 5 weeks. The ducks gained in all 108.75 pounds, and the chickens 22.5 pounds. How much more did the ducks gain than the chickens?

In these exercises answers are expected only to the nearest hundredth.

54. How many times as much did the ducks gain as the chickens?

55. How many pounds more, on the average, did 1 duck gain than 1 chicken?

56. How many pounds per week did the ducks gain? the chickens? How much more per week did the ducks gain than the chickens?

57. It cost $2.07 to feed the ducks for the whole period and $1.09 to feed the chickens. How much more did it cost to feed 1 duck than 1 chicken?

58. How many times as much did it cost to feed 1 duck as 1 chicken?

59. If it costs $2.07 to feed 39 ducks for 5 weeks, how much will it cost to feed 100 ducks for 1 week?

DENOMINATE NUMBERS

263. To ascertain the quantity of anything, or to *measure* it, is to find how many times it contains some established unit of the same kind, called the **unit of measure.**

Thus, to measure the corn in a bin is to find how many times the whole quantity of corn in the bin contains some unit measure, as 1 bushel, or 1 hundredweight.

A concrete number in which the unit of measure is established by law or custom is called a **denominate number.**

16 bushels is a denominate number; also 16 bushels 3 pecks.

264. A denominate number that is composed of units of one denomination only is called a **simple denominate number.**

16 bushels is a simple denominate number.

265. A denominate number that is composed of units of two or more denominations that are related to each other is called a **compound denominate number.**

16 bushels 3 pecks is a compound denominate number.

Tables of denominate numbers will be found on pages 278–280.

Reduction

266. Easy reductions.

EXERCISES

1. Reduce 72 inches to feet; to yards.
2. Reduce $1\frac{1}{2}$ yards to feet; to inches.
3. How many inches are there in 1 yd. 1 ft.?

Reduce:

4. $\frac{5}{8}$ lb. to ounces.
5. 1.4 T. to pounds.
6. .3 mile to rods.
7. $2\frac{1}{3}$ sq. yd. to sq. ft.
8. 18 cu. ft. to cu. yd.
9. $\frac{1}{4}$ day to minutes.
10. .75 gal. to pints.
11. $.12\frac{1}{2}$ bu. to quarts.
12. $5\frac{1}{2}$ hr. to minutes.
13. $7\frac{1}{2}°$ to minutes.

WRITTEN EXERCISES

267. **1.** Reduce 5 hr. 15 min. 12 sec. to seconds.

5 hr.
60
300
+ 15
315 min.
60
18900
+ 12
18912 sec.

Since there are 60 minutes in an hour, in 5 hours there are 5 × 60 minutes, or 300 minutes, and in 5 hr. 15 min. there are 300 minutes + 15 minutes, or 315 minutes.

Since there are 60 seconds in a minute, in 5 hr. 15 min. 12 sec., or in 315 min. 12 sec., there are 315 × 60 seconds, and 12 seconds more, or 18,912 seconds.

2. Reduce 18,912 seconds to hours, minutes, and seconds.

60 | 18912
60 | 315, + 12 sec.
5, + 15 min.
5 hr. 15 min. 12 sec.

Since 60 sec. = 1 min., 18,912 sec. = 315 min. and 12 sec. over.

Since 60 min. = 1 hr., 315 min. = 5 hr. and 15 min. over.

Hence 18,912 sec. = 5 hr. 15 min. 12 sec.

Reduce:

3. 4 wk. 3 da. to days.

4. 7 lb. 5 oz. to ounces.

5. 112 bu. 3 pk. to pecks.

6. 220 ft. 10 in. to inches.

7. 5 mi. 275 rd. to rods.

8. 14 yr. 7 mo. to months.

9. 3 yd. 2 ft. to inches.

10. 25 tons 12 cwt. to pounds.

11. 15 minutes to seconds.

12. 45 gal. 2 qt. to pints.

Reduce to compound denominate numbers:

13. 174 in.

14. 47 pt. (liquid).

15. 85 qt. (dry).

16. 21,929 sec.

17. 1924 oz.

18. 18,645 rd.

19. 7232 lb.

20. $33\frac{1}{2}$ pk.

21. $17\frac{3}{8}$ gal.

22. $85\frac{1}{4}$ min.

23. 1000 sq. in.

24. 1525 sq. ft.

25. 25,000 cu. in.

26. 12,300 cu. ft.

27. 104,965 cu. in.

28. The highest mountain in the world, Mt. Everest, is 29,002 feet high. What is its height in miles and feet?

29. I bought a 5-gallon can of maple sirup for $5. At what price per pint must I sell it to gain $1.40?

30. If it takes a pint of ice cream to serve 3 persons, how many persons will 4 gal. 3 qt. serve?

31. A dealer paid $4.75 for a crate containing 1 bu. 2 pk. of strawberries put up in quart boxes. If he sold them at 15¢ per box, how much did he gain?

32. When a train is running 45 miles an hour, what is its rate in feet per second?

33. When an automobile is running 44 feet per second, what is its rate in miles per hour?

34. The regulation military pace is a step 30 in. long, 2 steps per second. If a company of soldiers should keep this pace for 3 hr. 40 min., how many miles would they march?

35. The Park Row building, which contains 29 stories, is 309 feet high. Express in feet and inches the average height of a story.

268. More difficult reductions.

WRITTEN EXERCISES

1. What part of a bushel is 1 pk. 2 qt.?

SOLUTION

1 pk. 2 qt. = 10 qt.
1 bu. = 32 qt.
1 pk. 2 qt. = $\frac{10}{32}$ bu. = $\frac{5}{16}$ bu.

2. What decimal part of an hour is $40\frac{1}{2}$ min.?

SOLUTION

$40\frac{1}{2}$ min. = $\frac{40.5}{60}$ hr. = .675 hr.

3. Reduce 2° 33.′7 to seconds of arc.

SOLUTION

$$2° = 2 \times 3600'' = 7200''$$
$$33.'7 = 33.7 \times 60'' = 2022''$$
$$2° 33.'7 = 9222''$$

What part

4. Of a day is $4\frac{1}{2}$ hr.?
5. Of a yard is 1 ft. $1\frac{1}{2}$ in.?
6. Of a gallon is 2 qt. $1\frac{1}{2}$ pt.?
7. Of a bushel is 2 pk. 3 qt.?
8. Of a long ton is a short ton?
9. Of a cubic yard is 20 cu. ft. 36 cu. in.?

What decimal part

10. Of a mile is 1293.6 ft.?
11. Of a ton is 7 cwt. 64 lb.?
12. Of an hour is 1 min. 48 sec.?
13. Of a week is 2 days 10.8 hours?
14. Of a right angle is 4° 30′?
15. Of a square mile is 230.4 acres?

Reduce:

16. 28.5 yd. to inches.
17. 3.1875 bu. to quarts.
18. 2.428 tons to pounds.
19. .96875 of a mile to feet.
20. $\frac{5}{12}$ gal. to quarts and pints.
21. 39′ $22\frac{1}{2}''$ to a fraction of a degree.
22. $\frac{7}{8}$ of a right angle to degrees and minutes.
23. $\frac{5}{8}$ cu. yd. to cubic feet and cubic inches.
24. .9 of a day to a compound denominate number.

Addition and Subtraction

WRITTEN EXERCISES

269. **1.** Add 12 ft. 6 in. and 14 ft. 9 in.

ft.	in.
12	6
14	9
27	3

The sum of the inches is 15 in., or 1 ft. 3 in.; consequently 3 is written in the column for inches, and 1 is reserved to be united with the numbers in the column for feet.

The sum of the feet is 27 ft.

Hence, the entire sum is 27 ft. 3 in.

Add the following:

	ft.	in.
2.	8	4
	9	10

	yd.	in.
3.	4	24
	5	21

	gal.	qt.
4.	16	$3\frac{1}{2}$
	55	$2\frac{1}{2}$

	mi.	rd.
5.	22	100
	10	300

	lb.	oz.
6.	8	4
		13

	hr.	min.
7.	6	40
	8	50

	wk.	da.
8.	17	4
	12	3

	mo.	da.
9.	4	12
	3	14

	hr.	min.	sec.
10.	1	20	45
		39	15

	ft.	in.
11.	15	9
	6	10
	12	5

	yr.	mo.
12.	3	8
		11
	5	2

	T.	cwt.	lb.
13.	3	4	75
		9	45
	2	0	250

	cu. yd.	cu. ft.
14.	148	10
	352	15
	85	24
	124	5

	bu.	pk.	qt.
15.	20	1	4
	35	0	2
	15	7	0
	120	3	2

16.	44°	14′	48″
	36	27	16
	5	0	24
	4	18	12

17. Subtract 5 lb. 7 oz. from 21 lb. 2 oz.

lb.	oz.
21	2
5	7
15	11

Since 7 oz. cannot be subtracted from 2 oz., 1 lb. taken from the 21 lb. is reduced to ounces and united with the 2 oz., giving 18 oz. 18 oz. − 7 oz. = 11 oz., which is written in the remainder. Next subtracting 5 lb. from the 20 lb. left in the minuend, the number of pounds in the remainder is found to be 15.

Hence, the remainder is 15 lb. 11 oz.

Subtract:

	bu.	pk.
18.	25	2
	12	3

	ft.	in.
19.	100	$2\frac{2}{5}$
	50	4

	yd.	in.
20.	12	18
	9	27

	hr.	min.
21.	24	
	16	50

22.	180°	
	44′	45′

	T.	cwt.	lb.
23.	5	8	40
	2	3	160

	mi.	rd.
24.	4	80
	3	120

	mi.	ft.
25.	12	1000
	8	4500

	hr.	min.	sec.
26.	8	45	18
	6	30	56

27. How long was it from Apr. 10, 1899, to Feb. 2, 1906?

yr.	mo.	da.
1906	2	2
1899	4	10
6	9	22

The later date is written as the minuend and the earlier date as the subtrahend, writing the number of the month instead of its name.

Subtract as in denominate numbers, considering 30 days as a month and 12 months as a year. The remainder is the difference in time as accurately as it can be expressed in years, months, and days.

Subtract:

	yr.	mo.	da.
28.	1907	1	1
	1904	10	24

	yr.	mo.	da.
29.	1906	12	25
	1901	7	27

	hr.	min.	sec.
30.	16		
	14	45	10

31. How many years, months, and days old are you?

32. Find the age of each of the following men at the time when he became President of the United States:

NAME	BORN	BECAME PRESIDENT
George Washington	Feb. 22, 1732	Apr. 30, 1789
Abraham Lincoln	Feb. 12, 1809	Mar. 4, 1861
Ulysses S. Grant	Apr. 27, 1822	Mar. 4, 1869
Theodore Roosevelt	Oct. 27, 1858	Sept. 14, 1901

33. Add 25 ft. 6 in.; 32 ft. 4 in.; 28 ft. 8 in.; 56 ft. 10 in.; 18 ft. $11\frac{1}{2}$ in.; 23 ft. $4\frac{1}{2}$ in.

34. Add 5 hr. 3 min.; 16 hr. 10 min.; 4 hr. 45 min.; 7 hr. 59 min.; 6 hr. 17 min.; 14 hr. 11 min.

35. Subtract 22 bu. 3 pk. from 32 bu. $1\frac{1}{2}$ pk.

36. Subtract 6 dollars, 12 cents, 5 mills from 10 dollars.

37. Add 48 A. 37.5 sq. rd.; 72 A. 49.2 sq. rd.; 95 A. 85 sq. rd.; 56 A. 122.3 sq. rd.; 40 A. 140 sq. rd.

38. Add 88 rd. 7 ft.; 92 rd. $11\frac{1}{2}$ ft.; 16 rd. 14 ft.

39. A man left home at 9.30 A.M. on Thursday and returned at 5.45 P.M. on the Wednesday following. How long was he gone?

40. Some boys gathered 10 bushels of walnuts and sold all of them but 1 bu. 3 pk. Find the quantity sold.

41. The most northerly points reached by several of the more successful polar expeditions are as follows:

COMMANDER	NATIONALITY	YEAR	NORTH LATITUDE
Greeley	American	1882	83° 24′
Nansen	Norwegian	1896	86° 14′
Duke of Abruzzi	Italian	1900	86° 34′
Peary	American	1902	84° 17′

How near the north pole (90° north latitude) did each expedition reach? Express the distance in miles, taking $1' = 1.15$ mi.

Multiplication

WRITTEN EXERCISES

270. **1.** Multiply 16 bu. 3 pk. by 7.

bu.	pk.	
16	3	7 times 3 pk. = 21 pk. = 5 bu. 1 pk.
	7	7 times 16 bu. = 112 bu., and 112 bu. + 5 bu. = 117 bu.
117	1	The product is 117 bu. 1 pk.

Multiply:

2. 9 ft. 8 in. by 6

3. 7 gal. 3 qt. by 5

4. 2 hr. 9 min. by 8

5. 6 bu. 3 pk. by 4

6. 16° 35′ 20″ by 4

7. 11° 0′ 16″ by 15

8. 24 lb. 9 oz. by 20

9. 4 yr. 11 mo. by 11

10. Find the area, in square feet, of a rectangle 5 ft. 5 in. long and 2 ft. 6 in. wide.

$$\begin{array}{r} 65 \\ 30 \\ \hline 12\,|\,1950 \cdots \text{sq. in.} \\ 12\,|\,162.5 \\ \hline 13.542^{-} \cdots \text{sq. ft.} \end{array}$$

Since 5 ft. 5 in. = 65 in., and 2 ft. 6 in. = 30 in., the number of square inches in the area is 30×65, or 1950 sq. in.

Since there are 144 sq. in. in a square foot, the area in square feet may be obtained by dividing 1950 by 144, or by its factors 12 and 12, as shown.

The area is found to be 13.542 sq. ft., to the nearest third decimal place.

Find the area of each of the following, carrying inexact results to the nearest third decimal place:

	Figure	Base	Altitude	Area
11.	Rectangle	6 ft. 3 in.	4 ft. 7 in.	—— sq. ft.
12.	Rectangle	75 ft. $2\frac{1}{2}$ in.	32 ft. $4\frac{1}{2}$ in.	—— sq. ft.
13.	Triangle	44 ft. 1 in.	10 ft. 4 in.	—— sq. ft.
14.	Triangle	60 ft. 5 in.	33 ft. 11 in.	—— sq. ft.
15.	Square	18 ft. 6 in.	18 ft. 6 in.	—— sq. ft.
16.	Parallelogram	24 rd. 2 ft.	22 rd.	—— sq. ft.

17. Find the volume, in cubic feet, of a rectangular solid whose dimensions are 24 ft. 7 in., 12 ft. 11 in., and 5 ft. 4 in.

SUGGESTION. — First find the volume in cubic inches. Then, since there are 1728 cu. in. in a cubic foot, divide the number of cubic inches by 1728, or by $12 \times 12 \times 12$.

Find the volume, in cubic feet, of the following solids:

	LENGTH	BREADTH	THICKNESS
18.	19 ft. 4 in.	12 ft. 7 in.	6 ft. 2 in.
19.	28 ft. $3\frac{1}{2}$ in.	20 ft. 4 in.	5 ft.
20.	52 ft. $7\frac{1}{2}$ in.	33 ft. 6 in.	5 ft. 10 in.

Division

WRITTEN EXERCISES

271. 1. Divide 33 hr. 21 min. 45 sec. by 15.

	hr.	min.	sec.
15)	33	21	45
	2	13	27

33 hr. $\div$ 15 = 2 hr. and 3 hr. remainder.
3 hr. 21 min. = 201 min.
201 min. $\div$ 15 = 13 min. and 6 min. remainder.
6 min. 45 sec. = 405 sec.
405 sec. $\div$ 15 = 27 sec.

2. How many times is 2 hr. 13 min. 27 sec. contained in 33 hr. 21 min. 45 sec.?

SOLUTION

Reducing the denominate numbers to a *common denomination*, as seconds,

2 hr. 13 min. 27 sec. = 8007 sec.
33 hr. 21 min. 45 sec. = 120,105 sec.

Dividing 120,105 sec. by 8007 sec., the quotient is 15.
Hence 2 hr. 13 min. 27 sec. is contained 15 times in 33 hr. 21 min. 45 sec.

3. How many times is 8 ft. 6 in. contained in 144 ft. 6 in.?

SOLUTION

Reducing to a common denomination, in this case to feet,

8 ft. 6 in. = 8.5 ft. and 144 ft. 6 in. = 144.5 ft.

Dividing, the quotient is found to be 17.

Divide:

4. 39 gal. 1 qt. by 7
5. 81 lb. 9 oz. by 5
6. 53 ft. 6 in. by 6
7. 92 yd. 27 in. by 9
8. 202° 2′ by 11
9. 8 T. 250 lb. by 25
10. 102 mi. 120 rd. by 14
11. 280 sq. ft. 120 sq. in. by 30
12. 49 lb. 8 oz. by 5 lb. 8 oz.
13. 239 ft. 7 in. by 9 ft. 7 in.
14. 107° 36′ 6″ by 15° 22′ 18″

15. If the circumference of an automobile wheel is 8 ft. 3 in., how many revolutions will the wheel make in going a mile?

16. The distance around a square farm is 3 mi. 240 rd. Find the length of each side, in rods; the area in acres.

17. If a horse eats $1\frac{1}{2}$ pecks of oats per day, how long will $31\frac{1}{2}$ bushels of oats last him?

18. How long will 30 hundredweight of oats last him, if a bushel of oats weighs 32 pounds?

19. The area of a rectangle is 47 sq. ft. 36 sq. in. and the base is 8 ft. 9 in. Find the altitude.

SOLUTION

47 sq. ft. 36 sq. in. = 47.25 sq. ft.
8 ft. 9 in. = 8.75 ft.

Since 47.25 is the product of the number of feet in the base and the number of feet in the altitude, and the base is 8.75 ft., the number of feet in the altitude is 47.25 ÷ 8.75, or 5.4.

Hence, the altitude is 5.4 ft.

Find the missing dimension in each case:

20. Rectangle, area $38\frac{1}{4}$ sq. ft., base $8\frac{1}{2}$ ft.
21. Rectangle, area 1 acre, altitude $10\frac{2}{3}$ rd.
22. Triangular lot, area 10 acres, base $53\frac{1}{3}$ rd.

MISCELLANEOUS EXERCISES

272. **1.** A locomotive weighed 158,750 lb. Find the weight in tons and pounds.

2. A house was built on a lot 86 ft. 3 in. wide. If the width of the house was $\frac{1}{3}$ that of the lot, how wide was the house?

3. A stationer bought 9 half-gross boxes of pencils at $2.25 per box and sold the pencils at 5¢ apiece. Find his gain.

4. How much is paid for making 6 Persian rugs, each containing $4\frac{1}{3}$ sq. yd., if the workmen receive $3 per sq. yd.?

5. What is the average weight of the members of a football team, if the 11 men weigh together 1 ton 36 lb.?

6. At every beat the heart pumps 6 ounces of purified blood into the system. If a man's heart beats 70 times per minute, how many tons of purified blood leave the heart in one day?

7. If 1 lb. of India tea makes $7\frac{1}{2}$ gal. of liquid tea of a certain flavor and color, how much liquid tea will 4 oz. make?

8. If $721\frac{1}{2}$ lb. of wool are clipped from 74 sheep, what is the average clip per sheep?

9. An athlete ran 18 times around a track $293\frac{1}{3}$ yd. in circumference. How many miles did he run?

10. A stove dealer received by freight several shipments of stoves, weighing respectively: 1 T. 14 cwt. 64 lb., 3 T. 49 lb., 2 T. 9 cwt., 5 T. 18 cwt. 87 lb. Find the freight charges at $.25 per hundredweight.

11. When a grocer had sold $3\frac{7}{8}$ gal. of strained honey from a 5-gallon can at 36¢ per quart, he had received enough money to pay for the whole can. How much did he gain on the can?

12. A pumping engine at the waterworks worked from Mar. 1 to Sept. 1 without stopping day or night, and consumed 642 T. 1056 lb. of coal. How many pounds did it consume per hour?

PERCENTAGE

273. 1. If Margaret is given 200 words to spell and she misses 5 words out of every *hundred*, how many words does she miss?

2. A certain school has 400 pupils. One stormy day 7 per hundred, or 7 *per cent*, of the pupils were absent. How many were absent?

3. If a man's yearly salary is \$800 and he pays $\frac{12}{100}$, or 12 per cent, of it for house rent, how much is his rent?

4. Helen's book contains 300 pages. She read .15, or 15 per cent, of it one evening. How many pages did she read?

5. Find .06, or 6 per cent, of \$500; 2 per cent of \$700; 4 per cent of \$600; 8 per cent of \$900.

274. **Per cent** means *per hundred*, or **hundredths**, and the sign for it is %.

5 per cent, 5%, $\frac{5}{100}$, and .05 *represent the same thing.*

WRITTEN EXERCISES

275. Express as a common fraction:

1. 1%	4. 11%	7. 27%	10. 63%
2. 7%	5. 19%	8. 33%	11. 81%
3. 9%	6. 17%	9. 51%	12. 77%

Express as a decimal:

13. 15%	15. 36%	17. 4%	19. 10%
14. 24%	16. 45%	18. 6%	20. 30%

Express as per cent with the sign:

21. $\frac{9}{100}$	24. .48	27. .08	30. .60
22. $\frac{13}{100}$	25. .75	28. .20	31. .09
23. $\frac{41}{100}$	26. .55	29. .02	32. .90

276. Finding a per cent of a number.

1. Find $\frac{1}{3}$ of 12; $\frac{3}{4}$ of 16; $\frac{2}{5}$ of 20; $\frac{5}{6}$ of 24; $\frac{7}{8}$ of 32; $\frac{3}{10}$ of 80.

How do you find a fractional part of a number?

2. How many are $\frac{1}{100}$ of 300? 1% of 300? $\frac{7}{100}$ of 500? 7% of 500? $\frac{11}{100}$ of 600? 11% of 600?

3. Find .4 of 200; .04 of 200; 4% of 200; .08 of 500; 8% of 500; .12 of 300; 12% of 300.

4. How many are 3% of 700? 6% of 400? 2% of 1200?

5. Find 5% of \$300; 4% of \$400; 7% of \$900.

EXERCISES

277. How many are

1. $\frac{3}{100}$ of 400?
2. $\frac{9}{100}$ of 800?
3. $\frac{13}{100}$ of 200?
4. .07 of 700?
5. .16 of 300?
6. .21 of 400?
7. 6% of 900?
8. 5% of 1000?
9. 8% of 1200?

Find:

10. 10% of \$30
11. 20% of \$80
12. 11% of \$500
13. 14% of \$300
14. 25% of 400 bu.
15. 40% of 150 lb.

16. Mr. Livingston had \$100 and paid 22% of it for an overcoat. How much did the overcoat cost?

17. A farmer had 300 chickens, of which 15% were Plymouth Rocks. How many Plymouth Rocks had he?

18. There are 800 books in a library, of which 12% are historical. How many historical books are in the library?

19. In an orchard of 1100 trees 3% were plum trees. How many plum trees were there?

20. There are 200 children in a school, and 53% of them are girls. How many girls are there in the school?

21. A certain state senate has 50 members. When 30% of them are Democrats, how many Democratic senators are there?

WRITTEN EXERCISES

278. **1.** Find 24% of $47.25.

$47.25
.24
18900
9450
$11.3400

Since 24% of a number is .24 of it, 24% of $47.25 is .24 of $47.25.
Multiplying $47.25 by .24, the result is $11.34.
Hence 24% of $47.25 = $11.34.

Find:

2. 14% of $375
3. 26% of $562
4. 5% of $83.40
5. 7% of $60.86
6. 36% of $37.50
7. 85% of $84.26
8. 6% of $264.50
9. 8% of $409.25
10. 12% of $4500
11. 44% of $7084
12. 75% of $342.34
13. 92% of $620.72

14. Only 93% of a bushel of clover seed was good seed. If a bushel weighs 60 pounds, how many pounds of good seed were there?

15. How much did a grocer pay for a tub of butter that sold for $15, if it cost 88% of the sum for which it sold?

16. Of the 1380 persons on board an Atlantic steamship, the crew was 40%. How many were in the crew?

17. A vessel that has 3255 miles to go covers 14% of the distance in one day. Find the distance traveled in a day.

18. A crop of sweet potatoes sold for $743.75. If the expense of raising them was 48% of this, how much did the crop cost?

19. The wheat necessary to make a barrel of flour that sold at $4.75 was purchased for 68% of the value of the flour. How much was paid for the wheat?

20. A dealer was obliged to sell a lot of pineapples for which he had paid $115.50, at 92% of the cost. For how much did he sell them?

WRITTEN EXERCISES

279. 1. Express decimally and as a common fraction in its lowest terms: 25%; 5%; $33\frac{1}{3}\%$; 125%; $\frac{1}{2}\%$.

SOLUTIONS. $25\% = .25$; also $25\% = \frac{25}{100} = \frac{1}{4}$.

$5\% = .05$; also $5\% = \frac{5}{100} = \frac{1}{20}$.

$33\frac{1}{3}\% = .33\frac{1}{3}$; also $33\frac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3}$.

$125\% = 1.25$; also $125\% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$.

$\frac{1}{2}\% = .00\frac{1}{2}$; also $\frac{1}{2}\% = \frac{\frac{1}{2}}{100} = \frac{1}{200}$.

It is seen that $\frac{1}{2}\%$ means $\frac{1}{2}$ of 1%. What does $\frac{1}{4}\%$ mean? $\frac{2}{5}\%$? $\frac{3}{8}\%$?

Express decimally as hundredths:

2. $22\frac{1}{2}\%$	**5.** $3\frac{1}{3}\%$	**8.** $\frac{1}{5}\%$	**11.** 106%
3. $35\frac{3}{4}\%$	**6.** $6\frac{1}{4}\%$	**9.** $\frac{3}{4}\%$	**12.** 120%
4. $46\frac{2}{3}\%$	**7.** $8\frac{1}{8}\%$	**10.** $\frac{5}{8}\%$	**13.** 150%

Express as a common fraction in its lowest terms:

14. 20%	**19.** $12\frac{1}{2}\%$	**24.** $3\frac{1}{3}\%$	**29.** $83\frac{1}{3}\%$
15. 40%	**20.** $37\frac{1}{2}\%$	**25.** $6\frac{2}{3}\%$	**30.** $112\frac{1}{2}\%$
16. 50%	**21.** $62\frac{1}{2}\%$	**26.** $6\frac{1}{4}\%$	**31.** $133\frac{1}{3}\%$
17. 60%	**22.** $87\frac{1}{2}\%$	**27.** $8\frac{1}{3}\%$	**32.** $137\frac{1}{2}\%$
18. 75%	**23.** $66\frac{2}{3}\%$	**28.** $16\frac{2}{3}\%$	**33.** $266\frac{2}{3}\%$

34. To what per cent is $\frac{3}{4}$ equivalent? $\frac{1}{25}$? $\frac{2}{3}$? $1\frac{1}{5}$?

SOLUTIONS. $\frac{3}{4} = .75 = 75\%$ $\qquad \frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{1}{25} = .04 = 4\%$ $\qquad 1\frac{1}{5} = 1.20 = 120\%$

Find the per cent equivalent of:

35. $\frac{1}{2}$	**38.** $\frac{1}{3}$	**41.** $\frac{2}{3}$	**44.** $1\frac{3}{4}$	**47.** $\frac{5}{12}$
36. $\frac{1}{4}$	**39.** $\frac{3}{8}$	**42.** $\frac{5}{6}$	**45.** $1\frac{5}{8}$	**48.** $\frac{11}{16}$
37. $\frac{4}{5}$	**40.** $\frac{1}{6}$	**43.** $\frac{7}{8}$	**46.** $2\frac{1}{3}$	**49.** $\frac{32}{125}$

280. In many examples, when the *per cent* has an easy *fractional equivalent*, the work can be shortened by using the fraction.

Thus, in finding $33\frac{1}{3}\%$ of 36 it is much briefer and easier to do it by finding $\frac{1}{3}$ of 36 than by multiplying 36 by $.33\frac{1}{3}$.

The following *table of equivalents* should be *memorized*, and applied whenever possible:

TABLE

$50\% = \frac{1}{2}$	$60\% = \frac{3}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$
$25\% = \frac{1}{4}$	$80\% = \frac{4}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$
$75\% = \frac{3}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$20\% = \frac{1}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$40\% = \frac{2}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

EXERCISES

281. Find:

1. 50% of 18
2. 25% of 28
3. 20% of 25
4. 40% of 60
5. 75% of 48
6. $12\frac{1}{2}\%$ of 56
7. $33\frac{1}{3}\%$ of 45
8. $16\frac{2}{3}\%$ of 60
9. $37\frac{1}{2}\%$ of 72
10. $66\frac{2}{3}\%$ of 36
11. 75% of 80
12. $62\frac{1}{2}\%$ of 64
13. $66\frac{2}{3}\%$ of 48
14. $83\frac{1}{3}\%$ of 66
15. $87\frac{1}{2}\%$ of 96

16. What is 100% of 25? of 32? of 46? of any number? Find 200% of 25; 300% of 16; 500% of 12.

17. The Simplon tunnel in Switzerland is 12 miles long, and the St. Gothard is 75% as long. How long is the St. Gothard tunnel?

18. A man's salary was increased to \$24 per week. If his former salary was $87\frac{1}{2}\%$ of this amount, how much had he been receiving?

19. In a football game one team scored 18 points and the other $33\frac{1}{3}\%$ of that number. What was the score?

20. A lamb that weighed 90 pounds when sold had gained $16\frac{2}{3}\%$ of its weight in the previous two months. How much had it gained?

21. A certain ore yielded $66\frac{2}{3}\%$ of iron. How many tons of iron were obtained from 48 tons of the ore?

22. If an electric railway connecting two cities is 24 miles long and $37\frac{1}{2}\%$ of its length is within city limits, how many miles are within city limits?

WRITTEN EXERCISES

282. 1. Find $12\frac{1}{2}\%$ of \$275.20; $\frac{1}{8}\%$ of \$275.20.

$8)\overline{\$275.20}$
$\$ \ 34.40$

Since $12\frac{1}{2}\% = \frac{1}{8}$, the shortest way to find $12\frac{1}{2}\%$ of \$275.20 is to find $\frac{1}{8}$ of \$275.20, which is \$34.40.

Hence, $12\frac{1}{2}\%$ of \$275.20 = \$34.40.

$8)\overline{\$2.7520}$
$\$ \ .344\emptyset$

Since $\frac{1}{8}\%$ means $\frac{1}{8}$ of 1%, we first find 1% of \$275.20 by moving the decimal point two places to the left, and then find $\frac{1}{8}$ of \$2.7520 = \$.344.

Find:

2. 25% of \$93.84
3. $33\frac{1}{3}\%$ of \$46.08
4. $37\frac{1}{2}\%$ of \$91.36
5. $16\frac{2}{3}\%$ of \$79.74
6. $66\frac{2}{3}\%$ of \$84.42
7. $\frac{1}{4}\%$ of \$369.28
8. $\frac{1}{3}\%$ of \$878.91
9. $\frac{3}{8}\%$ of \$40,000
10. $\frac{1}{6}\%$ of \$39,762
11. $\frac{2}{3}\%$ of \$87,531
12. 120% of \$624.50
13. 250% of \$316.48
14. 175% of \$546.84
15. $162\frac{1}{2}\%$ of \$74.24
16. $166\frac{2}{3}\%$ of \$97.89

17. If a codfish trawl line contains 750 hooks, and the average catch fills 80% of the hooks, how many fish are taken each time?

18. The cost of building roads in Pennsylvania is divided as follows: state, $66\frac{2}{3}\%$; county, $16\frac{2}{3}\%$; town, $16\frac{2}{3}\%$. How much did each contribute toward a road that cost \$5685.30?

19. If artificial ice is made at a cost of \$1.60 per ton and retailed for $267\frac{1}{2}\%$ of the cost, what is the selling price?

20. There are 1528 people in a certain village in Ohio. If $87\frac{1}{2}\%$ of the inhabitants are native born, how many are native born?

21. If the income one year from a certain publication was \$14,568, of which $37\frac{1}{2}\%$ came from advertisements and $33\frac{1}{3}\%$ from sales and subscriptions, what was the income from each of these departments?

22. An acre of land produced 18,756 pounds of sugar beets. How many pounds of sugar did these beets contain, if a test showed them to be $16\frac{2}{3}\%$ sugar?

Only 75% of the sugar could be extracted. How many pounds of sugar did the acre yield?

MISCELLANEOUS EXERCISES

283. Find:

1. 6% of 243	**6.** 25% of 760	**11.** 105% of \$213.81
2. $3\frac{3}{4}\%$ of 368	**7.** $13\frac{1}{2}\%$ of 984	**12.** 110% of \$496.28
3. 9% of 785	**8.** 44% of 841	**13.** $137\frac{1}{2}\%$ of \$620.80
4. $\frac{5}{8}\%$ of 904	**9.** $16\frac{2}{3}\%$ of 549	**14.** 225% of \$391.04
5. $6\frac{1}{4}\%$ of 682	**10.** $87\frac{1}{2}\%$ of 968	**15.** $308\frac{1}{3}\%$ of \$845.37

16. A diver's suit weighs 125 pounds. How much does the helmet weigh, if it is 64% of the entire weight?

17. The total yield of some date palms was 128 bunches of dates. Find the average yield per tree, if it was $6\frac{1}{4}\%$ of the total yield.

One of these bunches weighed 45 pounds, but the average weight was 40% of this. What was the average weight per bunch?

18. It cost Mr. James \$196.50 to raise and market his cherries. If he paid 30% of this for picking them, how much did the picking cost?

If he sold the cherries for $283\frac{1}{3}\%$ of their cost, how much did he receive for them?

19. A car load of lambs weighing 25,000 pounds lost $7\frac{1}{2}\%$ of their weight in the course of transportation to market. Find the whole loss of weight.

20. Find the output of a starch factory that uses 1200 bushels of potatoes per day, if the potatoes weigh 60 pounds per bushel and yield 16% of their weight in starch.

284. Finding what per cent one number is of another.

1. What part of 100 is 31? What per cent of 100 is 31? What per cent of 100 is 5? 20? 25? 50? 75? $33\frac{1}{3}$?

2. What part of 10 is 5? How many hundredths of 10 is 5? What per cent of 10 is 5?

3. What part of 12 is 3? How many hundredths of 12 is 3? What per cent of 12 is 3? What per cent of 12 is 6? 9?

4. What part of 15 is 5? What per cent of a number is $\frac{1}{3}$ of it? What per cent of 15 is 5? 10?

5. What part of $\frac{4}{5}$ is $\frac{2}{5}$? What per cent of $\frac{4}{5}$ is $\frac{2}{5}$?

EXERCISES

285. What per cent of

1. 18 is 9?	**6.** \$36 is \$9?	**11.** $\frac{2}{3}$ is $\frac{1}{3}$?
2. 28 is 7?	**7.** \$48 is \$12?	**12.** $\frac{5}{8}$ is $\frac{3}{8}$?
3. 50 is 5?	**8.** 72 mi. is 9 mi.?	**13.** $\frac{3}{4}$ is $\frac{1}{2}$?
4. 60 is 12?	**9.** 25 bu. is 10 bu.?	**14.** $\frac{2}{3}$ is $\frac{1}{4}$?
5. 75 is 25?	**10.** 60 lb. is 15 lb.?	**15.** $\frac{3}{4}$ is $\frac{1}{8}$?

16. A quart is what per cent of a gallon?

17. What per cent of a pound is 12 ounces?

18. What per cent of an hour is 20 minutes?

19. Mrs. Hicks used 50 yards of thread from a 200-yard spool. What per cent of the spool of thread did she use?

20. Mr. Carter had \$24 in his pocketbook and spent \$6 for a ton of coal. What per cent of his money did he spend?

21. If 5 out of every 50 tons of anthracite coal mined in Pennsylvania are used about the mines, what per cent of the output is used there?

22. From an Alabama farm worked on shares 36 bales of cotton were secured, of which 12 went to the owner. What per cent of the cotton did the owner receive?

23. Of the 40 passengers on an electric car running between two cities, 30 rode the entire distance. What per cent of the passengers rode all the way?

24. A freight car and its load weighed 60 tons. If the load weighed 40 tons, what per cent of the total weight was the weight of the load?

WRITTEN EXERCISES

286. 1. What per cent of \$288 is \$14.40?

SOLUTION. — Since \$14.40 ÷ \$288 = .05, \$14.40 is 5 *hundredths*, or 5%, of \$288.

2. What per cent of \$8.64 is \$3.24?

SOLUTION. \$3.24 ÷ \$8.64 = .375; that is, \$3.24 is 37½% of \$8.64.

What per cent of

3. \$150 is \$72?

4. \$380 is \$95?

5. \$728 is \$455?

6. \$5.82 is \$1.94?

7. \$2.50 is \$4.05?

8. \$2160 is \$54?

9. \$77.25 is \$6.18?

10. \$48.32 is \$3.02?

11. \$50.94 is \$84.90?

12. \$85.36 is \$10.67?

13. $\frac{5}{6}$ is $\frac{5}{8}$?

14. $\frac{15}{16}$ is $\frac{9}{10}$?

15. .64 is .12?

16. .48 is .08?

17. 96% is 30%?

What per cent of

18. 1 mile is 52 rods?

19. 1 section is 100 acres?

20. 1 ton is 910 pounds?

21. 1 day is 1260 minutes?

22. If the income from an acre of strawberries was \$495 and the cost of production \$198, what per cent of the income was the cost?

23. What per cent of the 800 paper mills of this country does New York State contain, if it contains 180 paper mills?

24. In a certain school there are 752 pupils, of whom 282 are in the high school department. What per cent of the pupils are in the high school?

25. In crossing the Atlantic a vessel used 3570 of the 4760 tons of coal on board. What per cent of the supply was used?

26. A man who owed $20,000 could pay only $4900. What per cent of his debts could he pay?

27. Of 6225 baskets of fruit shipped from Delaware to New York, 2739 were spoiled by a delay. What per cent of the fruit spoiled?

28. A certain mining town contains 4160 inhabitants, of whom 3640 are miners. What per cent of the population are miners?

29. In a recent year there were 210,000 miles of ocean cable in the world, of which 40,600 were in the Atlantic Ocean. What was the per cent of cable in the Atlantic?

30. A 20-dollar gold coin was so badly worn that it had lost 10 cents in value. What per cent of its weight had been worn away?

31. In a year when there were 270,000 Indians in the United States, 60,000 of them could speak English. What per cent of the Indian population could speak English?

32. The United States one year produced 20,600,000 barrels of salt, of which Michigan furnished 7,725,000 barrels. What per cent of the whole did Michigan produce?

33. A manufacturing business valued at $225,000 is owned by three partners: Hale, Chesley, and Hoit. If Mr. Hale's share is $140,625, Mr. Chesley's $56,250, and Mr. Hoit's $28,125, what per cent of the business does each man own?

287. Finding a number when a per cent of it is given.

1. If $\frac{3}{4}$ of a number is 9, what is $\frac{1}{4}$ of it? What is the number?

2. If $\frac{7}{100}$ of a number is 21, what is $\frac{1}{100}$ of it? $\frac{100}{100}$ of it? If 7 % of a number is 21, what is 1 % of it? 100 % of it?

3. If 15 % of a number is 30, what is 1 % of it? What is the number? How does this result compare with $30 \div .15$?

In what two ways, then, may we find a number, if 15 % of it is given?

4. 6 % of a number is 24. Find the number in two ways.

5. If $33\frac{1}{3}$ %, or $\frac{1}{3}$, of a number is 4, what is the number? If 75 %, or $\frac{3}{4}$, of a number is 15, what is the number?

EXERCISES

288. Find the number of which

1. 12 is 1 %	6. 12 is 20 %	11. 10 is $\frac{1}{2}$ %
2. 15 is 5 %	7. 22 is 25 %	12. 13 is $33\frac{1}{3}$ %
3. 27 is 9 %	8. 41 is 50 %	13. 20 is $16\frac{2}{3}$ %
4. 60 is 30 %	9. 80 is 40 %	14. 25 is $12\frac{1}{2}$ %
5. 36 is 12 %	10. 27 is 75 %	15. 50 is $66\frac{2}{3}$ %

16. If 50 % of a man's working day is $4\frac{1}{2}$ hours, how many hours per day does he work?

17. A baseball team lost 8 games, or 25 % of the total number of games played. How many games were played?

18. When a ring is 16 carats fine, it is $66\frac{2}{3}$ % pure gold. If the ring were pure gold, how many carats fine would it be?

19. How many sweet potato plants does it take to yield a barrel of sweet potatoes, if 25 plants yield 20 % of a barrel?

20. How much mail and express matter passes through the Grand Central Station, New York, each day, if $33\frac{1}{3}$ % of it, or 300 tons, is mail matter alone?

21. If a fast freight goes 20 % of its journey from New Orleans to Louisville in 10 hours, how long does the trip take?

22. If it takes 25 % of a man's salary to pay his rent, which is $15 a month, what is his salary?

23. When a grocer had sold 40 % of a case of eggs, 12 dozen had been sold. How many dozen eggs did the case hold?

24. A fruit dealer sold 37½ % of a box of grapefruit. How many did the box contain at first, if he sold 24?

25. Find the total weight of a bale of sponges from Florida, if the covering weighs 2 pounds, or 4 % of the total weight.

26. It requires 120 tons of wood pulp to make the paper necessary to supply a certain newspaper for one day. If this is 80 % of the weight of the paper, how many tons of paper are used per day?

WRITTEN EXERCISES

289. 1. Find the number of which 84 is 24 %.

SOLUTION. 24 % of the number = 84;
that is, .24 of the number = 84.
Hence the number = 84 ÷ .24 = 350.

2. Find the number of which 273 is 87½ %.

SOLUTION. Since 87½ % = ⅞, 273 is ⅞ of the number.
Since ⅞ of the number = 273,
⅛ of the number = ⅐ of 273, or 39.
Hence, the number = 8 times 39, or 312.

Or, to shorten the work, the number = $\frac{8}{\cancel{7}}$ of $\overset{39}{\cancel{273}}$ = 312.

Find the number of which

3. 225 is 15 %	**8.** $24.84 is 6 %	**13.** $ 1.14 is ¾ %
4. 176 is 20 %	**9.** $36.95 is 4 %	**14.** $ 4.20 is ⅝ %
5. 384 is 25 %	**10.** $43.47 is 42 %	**15.** $79.62 is 37½ %
6. 504 is 28 %	**11.** $93.60 is 64 %	**16.** $87.95 is 166⅔ %
7. 880 is 55 %	**12.** $68.22 is 75 %	**17.** $90.97 is 183⅓ %

18. A manufacturer received 500 pounds of yarn, which was 8 % of his order. How much yarn had he ordered?

19. How much steel was used in constructing the Chicago government building, if 12% of it, or 1200 tons, was used in the dome?

20. In six hours 6000 tons of coal were loaded on a vessel. If this was 80% of her load, how many tons did she carry?

21. A quantity of rice lost 20% of its weight in being hulled at the mill. If the hulls weighed 4752 pounds, what was the weight of the rough rice?

22. When 6875 tons of ice had been put into an ice house on Lake Erie, $62\frac{1}{2}\%$ of its capacity was occupied. Find the capacity of the ice house in tons.

23. If the shrinkage of a quantity of wheat in drying was 2% and amounted to 775 pounds, how much did the wheat weigh before drying?

24. If 32% of the people in a certain city are voters, and there are 40,176 voters, find the population of the city.

25. It is estimated that by skimming cream from milk only 80% of the butter is obtained. If the modern method of separating extracts all of the butter, what should be the yield of a cow that by the old method yielded 180 pounds of butter per year?

26. A grocer bought 65 dozen ears of green corn at 12¢ per dozen. If this was 75% of the sum for which he sold the corn, how much did he receive for it?

27. A barrel of flour weighing 196 pounds sold at the grocery for $3\frac{1}{2}$¢ per pound, which was 140% of the cost. Find the cost.

28. How many feet of barbed wire are there on a spool, if 12 feet is $\frac{1}{5}\%$ of it?

29. If the productive area of coal in the United States is 154,220 square miles, or 55% of the entire coal area, what is the extent of the coal fields in this country in square miles?

290. Summary.

You have learned how to find: a per cent of a number; what per cent one number is of another; and a number when a per cent of it is given.

291. The number of which a per cent is found is called the **base.**

292. The *number* of hundredths found is called the **rate,** or **rate per cent.**

293. The result obtained by finding a per cent of the base is called the **percentage.**

Thus, in 5 % of \$120 = \$6, \$120 is the *base*, 5 % is the *rate*, and \$6 is the *percentage.*

294. With the aid of these terms, we may now express in brief form the principles learned in the preceding pages:

The **percentage** *equals the base multiplied by the rate.*

The **rate** *equals the percentage divided by the base.*

The **base** *equals the percentage divided by the rate.*

WRITTEN EXERCISES

295. 1. Given the base 245, and the rate 8%; find the percentage.

2. If the percentage is \$5.50 and the base \$22, find the rate.

3. Find the base, if the percentage is \$75 and the rate 4%.

Fill blanks:

	BASE	RATE	PERCENTAGE
4.	\$249.36	$16\frac{2}{3}$ %	———
5.	———	78 %	\$412.62
6.	\$694.80	———	\$260.55
7.	\$848.25	106 %	———

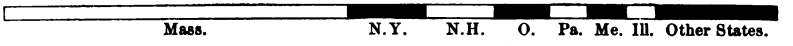
8. If \$275 will pay 55% of Mr. Hall's debts, how much does he owe?

9. Find the number of bushels of wheat in a schooner's cargo of 27,600 bushels of grain, if $46\frac{2}{3}$% of it is wheat.

10. What per cent of the work of painting a house did a painter do in $6\frac{5}{6}$ days, if he completed the work in $27\frac{1}{3}$ days?

11. If 28 shirts is $\frac{2}{5}$% of the weekly output of a shirt factory, how many shirts are made there each week?

12. A wholesale dealer bought 8270 barrels of sweet potatoes at \$1.25 per barrel. If he sold 60% of them @ \$1.75, 30% @ \$2.25, and the remainder @ \$1, how much did he gain?



13. This diagram, 4 inches long, represents the value of all the boots and shoes made in the United States in a certain year.

The part that represents the value produced by Massachusetts is 1.79 in. long. What per cent of the total value did Massachusetts produce that year?

14. New York's part of the diagram is .39 in. long; New Hampshire's, .37 in. long; Ohio's, .27 in. long; Pennsylvania's, .21 in. long; Maine's, .19 in. long; Illinois's .16 in. long.

What per cent did each of these states produce?

15. What per cent of the total value of the boots and shoes was produced in all the other states of the United States?

16. If 64% of the farms in the United States one year were worked by their owners, and there were 5,737,350 farms, how many were worked by their owners?

17. In a certain year 1,146,992 cords of wood were used in the manufacture of wood pulp. If this was 76% of all the wood used for pulp that year, how many cords were used?

Commercial Discount

296. 1. To stimulate trade a merchant reduced his prices 5% on purchases amounting to $20 or more. What was the reduction in the price of a rug marked $40?

2. If a manufacturer makes a deduction of 2% from his prices, for cash payment, what deduction for cash would he make on a bill of goods amounting to $300?

297. Any deduction made from a fixed price or amount is called a **discount.**

298. Discounts are usually reckoned as some per cent of the price or amount. The per cent of discount is called the **rate of discount.**

299. Many articles of merchandise have fixed prices that are published in catalogues and price lists.

Such prices are known as **list prices.**

300. For various reasons, discounts are often given from the list price or the marked price and are frequently spoken of as so many "per cent off."

Thus, if an article listed or marked at $1.50 is sold at "10% off," the discount is 10% of $1.50, or $.15, and the selling price $1.50 − $.15, or $1.35.

301. The price after the discount has been taken off is called the **net price.**

EXERCISES

302. Find the discount when the list price and rate of discount are:

1. 20¢, 5%	5. $12, 50%	9. $18, $33\frac{1}{3}$%
2. 50¢, 6%	6. $25, 20%	10. $64, $12\frac{1}{2}$%
3. 25¢, 8%	7. $36, 25%	11. $48, $16\frac{2}{3}$%
4. 80¢, 10%	8. $24, 40%	12. $72, $37\frac{1}{2}$%

13. Find the discount on a mandolin that is listed at \$25, if it is sold at 5% off for cash.

14. The list price of a wagon is \$80, from which 10% discount is offered. Find the net price.

15. At a bargain sale some tapestry curtains were marked down 20%. If the regular price was \$5 per pair, what was the selling price?

16. A bill of goods amounting to \$600 was discounted 3% for cash. How much was the discount?

17. A graphophone catalogued at \$50 was sold at 25% off. Find the discount.

18. How much does a lady pay for 4 yards of linen, if the price, \$.50 per yard, is marked down 8%?

19. To make room for new stock a dealer sold oilcloth marked \$.75 per yard at a discount of 20%. What was the selling price?

WRITTEN EXERCISES

303. Find the discount on articles listed as follows:

1. \$2.25, 8% off
2. \$3.80, 5% off
3. \$5.50, 6% off
4. \$14.75, 20% off
5. \$35.48, 25% off
6. \$54.75, $33\frac{1}{3}$% off

Find the net price, net cost, and total discount in each:

	Articles	Price	Rate of Discount
7.	12 tents	\$28.00	3 %
8.	15 rowboats	35.00	10 %
9.	20 shotguns	32.00	$37\frac{1}{2}$%
10.	18 gun cases	3.00	$16\frac{2}{3}$%
11.	24 revolvers	7.50	$33\frac{1}{3}$%
12.	36 fishing rods	1.60	$12\frac{1}{2}$%
13.	30 hunting knives	1.75	20 %

14. Find the net price of a desk listed at $32.50, discount 20 %.

15. There was a discount of 50 % on a set of Shakespeare's works the list price of which was $41.50. Find the discount.

16. The list price of a set of Dickens was $22.50. At a discount of 33⅓ %, for how much was the set sold?

17. If your father buys a set of books by Victor Hugo at $16.25, discount 40 %, how much does he pay?

18. A merchant sold go-carts at 30 % off. What was the selling price of one marked $18.50?

19. How much must a dealer pay for 9 cameras listed at $25 each, if there is a discount of 15 %?

20. Mr. Burton bought a suit of clothes from a dealer who was selling out, at 25 % off. If the suit was marked $27, how much did he pay for it?

21. Find the cash cost of 220 feet of garden hose at 7½¢ per foot, discount for cash 10 %.

22. Find the net cost of 3 cases of crayons, each holding 100 gross, at a discount of 20 %, list price 5¢ per gross.

23. Silk mufflers listed at $2.25 each are sold to a retailer at a discount of 16⅔ %. If the retailer buys 3 dozen, how much discount does he receive?

24. A manufacturer purchased 3600 yards of gingham at 6¾¢ per yard, and received 3 % discount by paying within 10 days. What was the net cost of the goods?

25. A hardware merchant ordered 250 pounds of horseshoe nails at $.22 per pound, discount 40 %. How much was his bill? By paying cash, the merchant received a further discount of 5 %. How much did he have to pay?

26. A merchant in Minneapolis bought a car load of damaged wheat weighing 42,000 lb. at 62½ % discount. If the original price was $.72 per bushel of 60 lb., find the net cost.

REVIEW PROBLEMS IN INDUSTRIES

304. 1. A man in Illinois raised 72 acres of broom corn. If $4\frac{1}{2}$ bu. of seed were required, how much seed was sown per acre?

2. He bought the seed by weight. If a bushel of seed weighs 50 pounds, how many pounds were there?

3. Find the cost of 225 lb. of seed at \$7.40 per 100 lb.

4. When the corn was full grown, the broom-making material, or brush, was obtained by cutting off the tops and thrashing out the seed. Find the cost of thrashing the crop from 72 acres at \$2.50 per acre.

5. To dry the brush it was placed on shelves 4 in. apart, in sheds 9 ft. 8 in. high. How many rows of shelves were there?

6. The cured brush was bound into 120 bales whose united weight was 20.7 tons. Find the average weight of a bale.

7. How many pounds of cured brush were produced per acre?

8. Find the yield of each acre in bales. What per cent of an acre's yield was used in making one bale?

9. It cost \$35 per ton to produce the crop and prepare the brush for market. What was the total expense?

10. The total cost being \$724.50, how much in all did the owner gain, if he sold the baled brush at $4\frac{3}{4}$¢ per pound?

11. Broom corn is often sold by the ton. How much more or less per ton would he have made, if he had sold it at $90 per ton?

12. If one ton of broom corn makes 100 dozen brooms of ordinary size, how many dozen brooms were produced from the crop of 20.7 tons? How many brooms did each of the 120 bales make?

13. In a recent year Illinois furnished 30,320 tons of broom corn, or $66\frac{2}{3}\%$ of the total amount grown in the United States. Find the production of broom corn in this country that year.

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14. In growing mushrooms, spawn is the material that takes the place of seed. Spawn consists of mushroom fibers in rich dirt. How many pieces of spawn were needed to plant a bed 120 feet long, if 4 pieces were planted for every 8 inches of its length?

15. A house for mushrooms had on the sides 6 such beds, and in the middle 4 beds, each of which held twice as much as one of the others. How many pieces of spawn had to be used?

16. If the spawn was bought in bricks, and each brick was broken into 12 pieces for planting, how many bricks were required for the 10,080 pieces?

17. How many pounds of spawn were planted, if each of the 840 bricks weighed $1\frac{1}{4}$ pounds? At $157.50 for the spawn, find its cost per pound.

18. If the bricks had been broken into 9 pieces instead of into 12, how many more bricks would have been used?

19. What would have been the extra cost for the 280 additional bricks at 15¢ per pound?

20. The yield of mushrooms was $\frac{3}{8}$ of a pound for every square foot of bed. How many pounds of mushrooms were produced, if the total area of the beds was 5040 sq. ft.?

21. The choice mushrooms, $33\frac{1}{3}\%$ of the crop of 1890 lb., were put into 10-pound baskets. How many baskets were used?

22. If 3-pound baskets were used for 50% of the mushrooms, find the number of 3-pound baskets used.

23. How many 5-pound baskets did the remainder fill?

24. The proprietor of the Laurel House bought the choice ones, 630 lb., at 65¢ per pound. How much did he pay for them?

25. The 3-pound baskets, 315 in all, were sold on the market for $378. What was the average price received for them per pound and per basket?

26. How much was received for the 315 pounds put up in 5-pound baskets at $2.25 per basket?

27. What were the receipts for the entire crop?

The scarcity of oysters in natural beds and the increased demand for oysters has led to the transplanting and raising of oysters either on old beds or on artificially prepared beds called "oyster farms." Where the ocean bottom is not naturally rocky or gravelly, it is prepared for the reception of the young oysters, or "seed oysters," by strewing it with oyster shells.

28. An oyster farm of 250 acres in Chesapeake Bay was

strewn with 17,500 bushels of oyster shells. Find their cost at $3\frac{1}{2}¢$ per bushel.

29. Thirty-five barrels of seed oysters from the Potomac were planted per acre. How many barrels of seed were required for the farm of 250 acres?

30. Find the cost of the seed, 8750 barrels, at $1.25 per barrel.

31. A barrel contains 15,000 seed oysters. How many oysters were planted per acre? How many were planted on the whole farm?

32. The oysters matured in three years. During this time 32% were destroyed by starfish and other natural enemies, and 6% stolen by "oyster pirates." How many of the 131,250,000 were thus lost?

33. Of the remaining 81,375,000, 84% were gathered. How many oysters were gathered?

34. Of the choice oysters, $3\frac{3}{4}$ millions were sold at $11¢$ a dozen, and $4\frac{1}{2}$ millions at $9¢$ a dozen. How much was received for them?

35. There were 15,000 barrels of second-grade oysters, averaging 1400 to the barrel. How much was received for these at $37\frac{1}{2}¢$ per hundred?

36. The rest of the oysters, 39,105,000, were shucked and sold at the rate of $\frac{1}{4}¢$ each. How much was received for them?

37. What were the total receipts for this crop of oysters?

38. A retailer bought a barrel containing 1368 second-grade oysters for $6.84. He removed them from the shells and sold them at 40¢ a quart. How much did he gain, if the average number in a quart was 48?

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39. A rice grower in South Carolina sowed 225 bushels of rice seed, or $2\frac{1}{2}$ bushels per acre. How many acres did he sow?

40. During the season the field received a depth of 18.72 inches of water from irrigating canals and 9.35 inches of rainfall. If 15.68 inches evaporated, find the useful depth of water.

41. After drying the field for harvest, the planter used two reapers to cut the 90 acres of grain. If each reaper cuts 11.25 acres per day, how long did it take to harvest the crop?

When the grain was thrashed there were 900 sacks of rough rice, weighing 162 pounds each. Find the number of:

42. Sacks per acre.

43. Bushels of 45 pounds each, per acre.

44. Bushels from the whole field.

45. For having his land irrigated the owner paid 2 sacks per acre to the canal company. What per cent of his crop did he pay? What per cent did he have left?

46. Having paid 20% of his crop of 3240 bushels for irrigation, how many bushels of rough rice did he have left?

47. This rice was sold at the mill for 85¢ per bushel. How much money was received for the 2592 bushels?

48. The expense, aside from irrigation, was $17.88 per acre. What was the expense for the 90 acres?

49. What was the profit on the crop?

50. How many pounds of rough rice were sold at the mill, each of the 2592 bushels weighing 45 pounds?

51. Every 162-pound sack of rough rice yielded 100 pounds of cleaned rice when milled. How many pounds of cleaned rice were obtained from the 116,640 pounds of rough rice?

52. How many hours did it take to mill this rice, if the capacity of the mill was 648 sacks per 12 hours?

53. How much less did the 72,000 pounds of cleaned rice weigh than the rough rice?

54. If the hulls removed from the rough rice weighed 23,328 pounds, what per cent of the rough rice consisted of hulls?

55. If the rest of the weight lost in milling (44,640 pounds) consisted of bran and polish which sold, when mixed, at $15 per ton, find the sum received for the bran and polish.

56. If 23% of the cleaned rice was broken during milling, how many pounds of broken rice were there? how many pounds of whole rice?

57. At $5\frac{3}{4}$¢ per pound, what was the value of the 55,440 pounds of unbroken rice?

58. If the 16,560 pounds of broken rice sold for $414, at what price per pound did it sell?

59. If Louisiana and Texas one year produced 297,900,000 of the 331,000,000 pounds raised in the United States, what per cent of the whole crop did these states produce?

60. At a cost of $1.05 per rod, find the expense of making $3\frac{1}{4}$ miles of irrigating canals for rice land.

61. A violet grower, in order to have a large number of cuttings from which to select, bought 14,550, and planted $\frac{1}{3}$ of them. How many cuttings did he plant?

62. His receipts during the season were distributed as follows:

October	$187.50
November	236.25
December	237.50
January	405.00
February	375.00
March	337.50
April	281.25
May	90.00

What per cent of the total receipts were the receipts of each month?

63. The schedule of prices per 100 violets was:

Oct. $1	Dec. $2	Feb. $1	Apr. $ .75
Nov. $1	Jan. $1.50	Mar. $ .75	May $ .75

How many violets were sold during each month? Find the total number sold.

64. Since 213,250 violets were obtained from 4850 plants, what was the average number of violets from each plant?

65. All these flowers were sent to florists in distant places at an express charge of 8¢ per 100 violets. Find the entire expense of sending them by express.

66. If these charges, $170.60, were $33\frac{1}{3}\%$ of the entire expense, how much did it cost to cultivate and market the violets?

67. What was the gain on the season's crop of violets?

MEASURES AND EQUIVALENTS

Measures of Weight

305. Avoirdupois weight.

The pound composed of 16 ounces, used in weighing bulky articles, is called an **avoirdupois pound.**

In delicate weighing, a small unit of weight called a **grain,** originally the weight of a grain of wheat, is used.

1 grain = $\frac{1}{7000}$ of an avoirdupois pound

For complete tables of denominate numbers see pages 278–280.

306. Troy weight.

In weighing gold, silver, platinum, and some kinds of jewels, a weight equal to 5760 grains, or $\frac{5760}{7000}$ of an avoirdupois pound, is used as a unit of weight.

This weight is called a **troy pound.**

$\frac{1}{12}$ of a troy pound, or 480 grains, is called a **troy ounce.**

$\frac{1}{20}$ of a troy ounce, or 24 grains, is called a **pennyweight.**

These related weights form the table of **troy weight.**

24 grains (gr.)	= 1 pennyweight,	pwt.
20 pennyweights	= 1 ounce,	oz.
12 ounces	= 1 pound,	lb.

In writing prescriptions physicians often use these units:

20 grains = 1 scruple
3 scruples = 1 dram
8 drams = 1 ounce (troy)

Except in filling prescriptions, druggists buy and sell by avoirdupois weight.

WRITTEN EXERCISES

307. 1. Which is heavier, and how much, a pound of gold or a pound of cotton?

2. How many grains are there in an avoirdupois ounce?

3. What decimal part of a troy ounce is an avoirdupois ounce? Obtain the answer to the nearest thousandth.

4. A druggist bought 1 100-ounce tin of quinine, 3 50-ounce tins, 5 25-ounce tins, and 3 15-ounce tins. How many pounds and ounces of quinine did he buy?

5. A drug company wishes to make 4000 pills containing 5 grains of quinine each, 4000 containing 3 grains each, and 12,000 containing 2 grains each. How many pounds of quinine must be used? Find its cost at $26\frac{1}{2}¢$ per ounce.

6. A man sold 125 bales of cotton, weighing 30 T. 875 lb. net, for 8.45¢ per pound. How much did he receive?

7. Find the value of 12 T. 840 lb. of pig iron, at $18.40 per long ton.

8. When oats are selling at $1.10 per 100 lb., how much will be received from a sale of 5 T. 635 lb.?

9. How much is a pound of gold worth at the United States coinage rate, $20.67183 per ounce?

Find the value, to the nearest cent, of the following weights of silver, at 66¢ per ounce:

10. 5 lb. 8 oz.
12. 4 oz. 5 pwt.
14. 2 lb. 10 oz. 3 pwt.
11. 2 lb. 10 oz.
13. 11 oz. 4 pwt.
15. 1 lb. 16 pwt. $12\frac{1}{2}$ gr.

16. The United States silver dollar weighs $412\frac{1}{2}$ grains, and is $\frac{9}{10}$ pure silver. Find its bullion value, or the value of the silver in it, when silver is worth 64 cents per ounce.

17. How many silver spoons, each weighing 2 oz. 5 pwt., can be made from a silver bar weighing 6 lb. 4 oz. 10 pwt.?

18. *A double eagle weighs* $21\frac{1}{2}$ *pennyweights.* United States gold coins are $\frac{9}{10}$ pure gold and $\frac{1}{10}$ copper, by weight. How many grains of pure gold does a double eagle contain?

19. The value of a gold coin is the value of the pure gold in it. The value of a double eagle is $20. How much, to the nearest hundredth of a cent, is 1 pennyweight of pure gold worth?

20. How many pennyweights of pure gold are worth $1?

21. A Newark goldsmith bought 1 lb. 2 oz. of gold bullion at $1.03 per pennyweight. How much did it cost him?

22. Gold coin in banks is often piled in bags containing $5000 each, and is weighed instead of being counted out. How many pounds and ounces should such a bag contain?

23. If you should draw $15,000 in gold coin from a bank, how many pounds and ounces of gold ought you to receive?

24. Gold coins are received by the government at their face value so long as the loss of weight due to wearing does not exceed $\frac{1}{2}\%$. What is the least weight of a double eagle that the government will receive as $20?

25. How much is a double eagle worth when it has lost $\frac{3}{4}\%$ in weight, if it is then worth only its actual weight?

26. Find the weight of $1,000,000 in United States gold coin fresh from the mint.

27. In shipping $1,000,000 in new gold eagles from New York to London, the loss in weight due to wear amounted to 188 pwt. 3 gr. Find the loss in dollars.

What per cent of the million dollars was lost?

28. Seventeen tons of ore sent to the smelter contained .48 oz. of gold worth $20 per ounce, and 25 oz. of silver worth 65¢ per ounce. Find the value of precious metal per ton of ore.

29. One year the total production of platinum in the United States was 7 lb. 10 oz., worth $1814. Find its value per ounce.

Measures of Volume and Capacity

308. A pile of 4-foot firewood, 8 feet long and 4 feet high, is called a **cord.**

1 cord = 128 cu. ft.

309. One fourth of a pint, liquid measure, is called a **gill.**

This measure is little used.

For the more common measures of volume and capacity, see the tables.

310. Useful equivalents.

A gallon is equivalent to 231 cu. in., and a bushel to 2150.42 cu. in. Since 2150.42 cu. in. is only about 10 cu. in. less than $1\frac{1}{4}$ cu. ft., it is sufficiently accurate for all practical purposes to take $1\frac{1}{4}$ cu. ft. as the equivalent of a bushel.

1 gallon = 231 cu. in.
1 bushel = $1\frac{1}{4}$ cu. ft.

WRITTEN EXERCISES

311. Find the number of cords of 4-foot wood in:

1. A pile 120 ft. long and 4 ft. high.
2. A pile 76 ft. long and 6 ft. high.
3. A pile $\frac{1}{4}$ mi. long and 8 ft. high.
4. A load 8 ft. long and $4\frac{1}{2}$ ft. high.
5. How many cubic inches are there in a quart, liquid measure? in a quart, dry measure?
6. How many bushels of grain can be stored in an elevator bin 7 ft. square and 80 ft. high?
7. A vat at a cheese factory is 11 ft. long, 4 ft. wide, and $3\frac{1}{2}$ ft. deep. How many gallons of milk will it contain?

8. The inside dimensions of a car are: length, 36 ft.; width, 8 ft. 4 in.; height, 8 ft. Find its cubical contents.

9. If the car could be filled to the top with oats weighing 32 pounds to the bushel, how many pounds of oats would it contain, the cubical contents of the car being 2400 cubic feet?

10. The greatest load that the railroad company allows the car to carry is 80,000 lb., called its *capacity*. Since the weight of the oats when the car is filled to the top is 61,440 lb., how much less is that than the permitted capacity of the car?

11. If the car could be loaded to the top with wheat weighing 60 pounds to the bushel, how much would the load exceed the capacity allowed by the company?

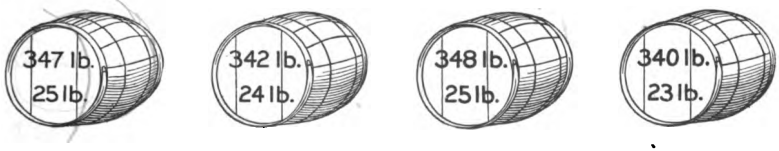
12. How many bushels of wheat would there be in the car, if the bottom were covered to a depth of 1 foot?

13. To what height is the car filled with wheat when the load is the greatest the car may carry, 80,000 pounds?

14. When empty, the car weighs 35,000 pounds. Find its weight when loaded with 1200 bushels of shelled corn weighing 56 pounds to the bushel.

15. Find its weight when loaded with 200 2-bushel bags of timothy seed, 60 pounds to the bushel, 80 200-pound bags of bran, and 225 bushels of potatoes, 60 pounds to the bushel.

16. At a grocery store a boy saw these marks on the heads of some barrels of sugar:



The first weight is the weight of barrel and sugar, the second the weight of the empty barrel. Find the cost of the sugar in all the barrels at $4.25 per 100 pounds.

The abbreviation for barrel or barrels is bbl.

Find the weight and cost of:

17. 16 bbl. flour, each 196 lb., at \$4.75 per barrel.

18. 20 bbl. salt, each 280 lb., at \$1.15 per barrel.

19. 120 bbl. pork, each 200 lb., at \$12.80 per barrel.

20. 2 bbl. dressed chickens, 275 lb., 280 lb., at $13\frac{1}{2}$¢ per pound.

21. 3 tierces lard, 250 lb., 290 lb., 260 lb., at \$7.55 per 100 lb.

Find the gain on each of the following:

	Article	Buying Price	Selling Price
22.	1 bbl. linseed oil, 49 gal.	38¢ per gal.	50¢ per gal.
23.	1 bbl. kerosene, 52 gal.	\$5.75	14¢ per gal.
24.	1 bbl. turpentine, 50 gal.	$67\frac{1}{2}$¢ per gal.	90¢ per gal.
25.	1 bbl. pine tar, 50 gal.	\$5.25	25¢ per gal.
26.	1 bbl. paint, 47 gal.	58¢ per gal.	75¢ per gal.
27.	1 bbl. molasses, 50 gal.	$28\frac{1}{2}$¢ per gal.	35¢ per gal.
28.	1 bbl. potatoes, $2\frac{3}{4}$ bu.	\$1.60	20¢ per pk.
29.	4 bags peanuts, each 4 bu.	\$1.40 per bu.	5¢ per qt.
30.	6 boxes dates, each 30 lb.	\$2.50 per box	10¢ per lb.
31.	7 cases eggs, each 30 doz.	\$6.45 per case	26¢ per doz.
32.	8 tubs butter, each 60 lb.	$22\frac{1}{2}$¢ per lb.	27¢ per lb.
33.	12 25-lb. boxes apricots.	$9\frac{1}{2}$¢ per lb.	12¢ per lb.
34.	18 tons meal, 10-lb. bags.	\$1.25 per bag	\$1.45 per bag
35.	12 tons bran, 200-lb. bags.	\$19 per ton	\$1.10 per cwt.

312. Water in large quantities is measured by the barrel of $31\frac{1}{2}$ gallons, by the cubic foot, and by the thousand or million gallons.

In all ordinary calculations the weight of a cubic foot of water is taken as $62\frac{1}{2}$ lb. or 1000 oz.

$$1 \text{ cu. ft.} = 1728 \text{ cu. in.} = \tfrac{1728}{231} \text{ gal.} = 7.48^{+} \text{ gal.}$$

Since $1 \text{ gal.} = 231 \text{ cu. in.} = \tfrac{231}{1728}$ cu. ft., the weight of 1 gallon of water weighing $62\frac{1}{2}$ lb. per cu. ft. is $\frac{231}{1728}$ of $62\frac{1}{2}$ lb., or 8.355^{+} lb.

WRITTEN EXERCISES

313. Hereafter, unless the contrary is expressly stated, use the following measures and approximate equivalents:

1 barrel of water = $31\frac{1}{2}$ gallons
1 cubic foot = $7\frac{1}{2}$ gallons
1 cubic foot of water weighs $62\frac{1}{2}$ pounds
1 gallon of water weighs $8\frac{1}{3}$ pounds

1. Find the capacity in barrels of a rectangular cistern 6 feet square and $10\frac{1}{2}$ feet deep; of one 16 feet deep, $8\frac{1}{2}$ feet long, and $7\frac{1}{2}$ feet wide.

2. The tank in the tender of a locomotive held 7500 gallons. Find its capacity in cubic feet; in barrels.

3. A city used $18\frac{3}{4}$ million gallons of water per day. Express this quantity of water in cubic feet.

4. A city had 8 slow sand filters for purifying its drinking water. The area of each was 30,000 sq. ft., and the flow of water through each was 110 in. per day. How many gallons of water were filtered per day?

5. The area drained by the Hudson River above Albany is about 8240 sq. mi. A square mile is equal to 27,878,400 sq. ft. How many million gallons of water, to the nearest million, fall on this area during a year if the total rainfall is 40 in.?

6. The water of the Dead Sea is 1.24 times as heavy as fresh water. How much does a gallon of Dead Sea water weigh?

7. Milk is about 1.03 times as heavy as water. What is the approximate weight of milk in a vat containing 600 gallons of milk?

8. Ice is .92 as heavy as water. How much will a block of ice 2 ft. by 30 in. by 12 in. weigh?

9. A cubic foot of one kind of marble weighed $163\frac{3}{4}$ pounds, and of another kind $184\frac{3}{8}$ pounds. How many times as heavy as water was each kind?

10. Cork is .24 as heavy as water. How much does a cubic foot of cork weigh?

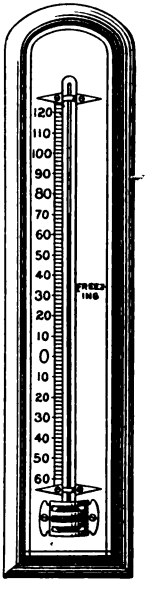
11. Find the weight of a granite corner stone 4 ft. square and 30 in. thick, if granite is 2.72 times as heavy as water.

12. Steel is about 7.84 times as heavy as water. Find the weight of a steel plate 90 in. by 56 in. and $\frac{3}{8}$ in. thick.

13. Lead is 11.35 times as heavy as water and tin 7.28 times as heavy as water. How much more does a cubic foot of lead weigh than a cubic foot of tin?

Measures of Temperature

314. On a Fahrenheit (F.) thermometer, the one in common use in America, the freezing point of water is marked 32, for 32 *degrees*, and the boiling point 212. For ordinary purposes, however, thermometers are not marked as far as the boiling point.



Since the difference between the freezing and boiling points of water, $212° - 32°$, is 180°, one **degree** Fahrenheit is $\frac{1}{180}$ of this difference.

Temperatures below 0° are written with a minus sign, thus: $-1°$, $-2°$, $-10°$, etc.

EXERCISES

315. **1.** At 5 A.M. the temperature was $-8°$ F. and at 2 P.M. it was 24° F. Find the rise in temperature.

2. At noon the temperature was 14° F. and at 10 P.M. it was $-3°$ F. What was the fall in temperature?

3. How far below the freezing point of water is 12° F.? — 12° F.?

4. During an ascent of Pike's Peak the temperature fell from 60° F. to 31° F. How many degrees did it fall?

5. When the temperature in Montreal is − 14° F. and in New Orleans 58° F., what is the difference in temperature between the two places?

6. The temperature of a beef carcass was reduced from 96° F. to 30° F., the temperature of cold storage. How many degrees was the temperature reduced?

7. In freezing ice cream a mixture of ice and salt reduced the temperature from 50° F. to 28° F. How many degrees was the temperature reduced?

WRITTEN EXERCISES

316. 1. Cast iron melts at 2100° F. and steel at 2500° F. How much higher is the melting point of steel than that of cast iron?

2. Tin melts at 442° F. How many degrees above the melting point of ice (32° F.) is this? how many degrees above the boiling point of water?

Find the difference between the highest and lowest temperature in the following cities during a recent year (Fahrenheit scale):

City	Highest	Lowest	City	Highest	Lowest
3. Havre, Mont.	108°	−55°	**8.** Portland, Me.	97°	−17°
4. Cincinnati	105°	−17°	**9.** Portland, Ore.	102°	− 2°
5. Duluth	99°	−41°	**10.** Boston	102°	−13°
6. Mobile	102°	− 1°	**11.** San Diego	101°	32°
7. Key West	100°	41°	**12.** Santa Fe	97°	−13°

13. How much more did the temperature in Duluth vary than that in Key West?

Lumber Measure

317. The number of **board feet** in a board 1 inch (or less) in thickness is the number of square feet of surface in one side of the board.

A board 1 foot wide and 15 feet long contains 15 square feet, or 15 feet *board measure*, if the board is 1 inch or less in thickness.

For brevity we often use the word "foot" instead of board foot, and "thousand feet" or M instead of thousand board feet.

318. *When the thickness is greater than 1 inch, the number of feet board measure is obtained by multiplying the number of feet in length by the number of feet in width, and this product by the number of inches in thickness.*

Thus, a board 6 ft. long and 10 in. wide contains $6 \times \frac{5}{6}$ ft., or 5 ft., if it is 1 in. thick or less than 1 in. thick; if it is 2 in. thick, it contains 2×5 ft., or 10 ft.; if $1\frac{1}{2}$ in. thick, $1\frac{1}{2} \times 5$ ft., or $7\frac{1}{2}$ ft.

The dimensions 6 ft. by 10 in. by 2 in. are commonly written in this way: $6' \times 10'' \times 2''$.

EXERCISES

319. Find the number of feet in boards 1 inch or less in thickness that have the following lengths and widths:

1. $4' \times 3''$	**10.** $9' \times 8''$	**19.** $18' \times 14''$
2. $16' \times 3''$	**11.** $24' \times 8''$	**20.** $18' \times 16''$
3. $22' \times 3''$	**12.** $7\frac{1}{2}' \times 8''$	**21.** $18' \times 18''$
4. $15' \times 4''$	**13.** $12' \times 10''$	**22.** $18' \times 20''$
5. $24' \times 4''$	**14.** $18' \times 10''$	**23.** $12' \times 22''$
6. $20' \times 4''$	**15.** $15' \times 10''$	**24.** $16' \times 24''$
7. $10' \times 6''$	**16.** $12' \times 12''$	**25.** $28' \times 24''$
8. $14' \times 6''$	**17.** $16' \times 12''$	**26.** $22' \times 30''$
9. $11' \times 6''$	**18.** $12' \times 14''$	**27.** $16' \times 36''$

28. How many board feet are there in a 2-inch plank $10\frac{1}{2}$ feet long and $2\frac{1}{4}$ feet wide?

WRITTEN EXERCISES

320. Find the number of feet in the following timbers:

	Number of Pieces	Size	Length	Number of Feet
1.	24	$2'' \times 12''$	$12'$	——
2.	15	$2'' \times 6''$	$14'$	——
3.	250	$2'' \times 4''$	$10'$	——
4.	112	$3'' \times 8''$	$16'$	——
5.	64	$2'' \times 8''$	$14'$	——
6.	80	$2'' \times 12''$	$12'$	——
7.	125	$4'' \times 6''$	$18'$	——
8.	75	$6'' \times 8''$	$18'$	——
9.	48	$4'' \times 8''$	$12'$	——
10.	84	$7'' \times 9''$	$11'$	——
11.	112	$6'' \times 14''$	$14'$	——
12.	96	$8'' \times 16''$	$20'$	——

13. Find the cost of each of these lots of timber at \$24 per M.

In an expression like "sills $4'' \times 8'' \times 200'$", $200'$ is the *aggregate* length of sills; in "7 posts $4'' \times 8'' \times 22'$", $22'$ is the length of *each* post.

Foot the following lumber bills at \$21 per M:

14.
Sills $4'' \times 8'' \times 200'$
Girder $8'' \times 12'' \times 30'$
7 posts $4'' \times 8'' \times 22'$
2 posts $4'' \times 8'' \times 18'$
Beams $3'' \times 8'' \times 84'$

15.
Plates $4'' \times 6'' \times 100'$
Ties $4'' \times 6'' \times 340'$
Ridge $3'' \times 12'' \times 28'$
45 beams $3'' \times 8'' \times 22'$
24 rafters $2'' \times 8'' \times 20'$

16.
Sills $6'' \times 8'' \times 250'$
Plates $4'' \times 6'' \times 250'$
Ties $4'' \times 6'' \times 325'$
Girders $8'' \times 12'' \times 46'$
10 posts $4'' \times 8'' \times 24'$

17.
70 joists $2'' \times 10'' \times 22'$
20 joists $2'' \times 10'' \times 15'$
8 joists $2'' \times 10'' \times 13'$
Purlins $4'' \times 6'' \times 240'$
Beams $3'' \times 8'' \times 200'$

Find the cost of:

18. 440 studs, each $2'' \times 4'' \times 12'$, at \$20 per M.

19. 320 spruce rafters, $2'' \times 10'' \times 18'$, at \$22 per M.

20. 8 spruce girders, $8'' \times 12'' \times 28'$, at \$28 per M.

21. 800 hemlock boards, $8''$ wide, $12'$ long, at \$17 per M.

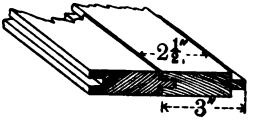
22. 640 lineal feet chestnut molding, at $2\frac{1}{2}¢$ per foot.

23. 6250 board feet maple flooring at \$36 per M.

24. 6 pine corner posts, $4'' \times 8'' \times 36'$, at \$30 per M.

25. At \$20 per thousand feet, what will be the cost of the boards for a fence 64 rods long and 5 boards high, each board being 6 in. wide and 1 in. thick?

26. How many pieces of Georgia pine flooring, $16'$ long and $2\frac{1}{2}''$ wide when laid, are required for the floor of a room $15' \times 16'$?



27. The boards are $\frac{7}{8}''$ thick. Since they were $3''$ wide before being tongued and grooved, they are sold as boards $3''$ wide. Find the cost of the flooring at \$54 per M.

28. The following materials were used in making a double floor:

38 spruce joists, $2'' \times 12'' \times 24'$, at \$25 per M.
160 hemlock boards for lining, $1'' \times 7'' \times 12'$, at \$16 per M.
38 strips, $2'' \times 2'' \times 24'$, at $2¢$ per lineal foot.
30 strips (for bridging), $2'' \times 3'' \times 15'$, at \$22 per M.
64 planed boards, $1\frac{1}{4}'' \times 9'' \times 26'$, at \$30 per M.
1 keg of nails, \$3.65.
Find the cost of all the materials mentioned.

29. A plank of Oregon fir, sent to the New York State Commissioner of Education to be made into a table top, measured 24 ft. by 5 ft. by 7 in. How many feet of lumber did it contain?

30. It weighed 2050 lb. Find its weight per cubic foot.

Plastering, Painting, and Kalsomining

321. Plastering, painting, and kalsomining are often computed by the square yard.

Laths are 4 feet long and are sold in bundles of 50 or 100.

A bundle of 100 laths will cover 5 square yards of wall.

In the following exercises 1 bundle means 100 laths.

WRITTEN EXERCISES

1. The cost of 100 square yards of plastering was estimated by a plasterer to be as follows:

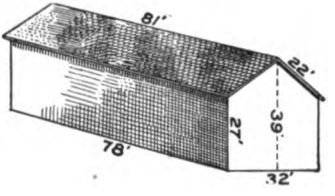
2000 lath at $2.25 per M	$ 4	50
Labor of putting on lath at 10¢ per bundle		
10 lb. of lath nails @ 6¢		
5 loads of sand @ 90¢		
4 barrels of common lime @ $1		
1 barrel of finishing lime @ $1.50	1	50
1½ bu. of plaster @ 25¢		
1 bu. of hair @ 50¢		50
Mason, 1½ days @ $3.50		
Mason's helper, 1½ days @ $2		
Laborer, ½ day @ $1.50		
Use of screen for sand, etc.	1	
Cartage	2	20
Profit, 10% of the above		
Total cost of 100 sq. yd.	$	

Find the cost of plastering per square yard.

2. Find the cost, at 35¢ per square yard, of plastering the walls and ceiling of a room 16 ft. by 14 ft. and 9 ft. high, deducting the area of 2 windows 3½ ft. by 6 ft. and of 2 doors 3 ft. by 7 ft.

3. Find the cost of painting the walls of the room mentioned in exercise 2 at 30¢ per square yard (deducting for windows and doors), and of tinting the ceiling at 10¢ per square yard.

4. Find the cost of painting the sides and ends of this barn, at $12\frac{1}{2}$ ¢ per square yard.



5. Find the cost of painting the roof of the barn at 16 ¢ per square yard.

Find the cost of plastering each of the following rooms, each 10 feet high, at 32 ¢ per square yard, deducting 1.2 sq. yd. for each door and .9 sq. yd. for each window:

6. Parlor 17′ 6″ × 16′, 3 doors, 4 windows.
7. Dining room 14′ 6″ × 14′, 4 doors, 3 windows.
8. Kitchen 11′ 1″ × 14′, 4 doors, 1 window.
9. 2 bedrooms, each 13′ 1″ × 10′, 2 doors, 1 window.
10. 1 bedroom 8′ 7″ × 9′, 1 door, 1 window.
11. Bathroom 8′ 7″ × 5′ 8″, 1 door, 1 window.
12. Hall 3′ 7″ × 20′ 4″, 8 doors.
13. Hall 3′ 7″ × 15′ 9″, 3 doors, 1 window, 1 skylight 6′ × 8′.
14. How many bundles of lath, to the nearest bundle, will be required to cover the walls and ceiling of a room 18 ft. by 14 ft. and 9 ft. high, making no deductions for openings?
15. Find the cost of painting the front of an apartment house 28 feet wide and 54 feet high, at 20 ¢ per square yard, deducting the area of 14 windows each 4′ × 7′, and of one door 5′ × 9′.

Roofing

322. An area of 100 square feet is called a **square**.

Common shingles are about 18 inches long and average 4 inches in width. When laid "4 inches to the weather," the exposed surface of 1 shingle is 4 × 4 sq. in., or $\frac{1}{9}$ sq. ft.

It takes 900 shingles, then, to cover 1 square, but to allow for waste 1000 shingles per square are often estimated.

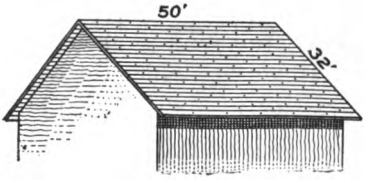
WRITTEN EXERCISES

323. **1.** How many bunches of shingles, 250 shingles to the bunch, are required to shingle the roof of the barn shown on the preceding page, estimating 1000 shingles per square?

The fractional part of a bunch cannot be bought.

2. Find the cost of the shingles, 143 bunches, at $2.20 per M.

3. How much would it cost to shingle the roof shown here, at $3.20 per M for cedar shingles, $7.50 for shingle nails, and $16 for labor?



4. How much would it cost to cover the roof with galvanized sheet steel at $4.75 per square?

5. A cheaper roof may be made by covering the sheathing boards, shown in the picture, with sheathing paper, and covering this paper with felt roofing properly nailed and cemented. If the paper is 36 in. wide and is laid on lengthwise of the roof, with a lap of 4 in., how many strips will be required?

6. If 3 strips can be cut from each roll of sheathing paper, how much will the 24 strips of paper cost at 60 ¢ per roll?

7. Find the cost of the felt roofing at $87\frac{1}{2}$ ¢ per roll, if each roll covers one square.

8. Find the whole cost of this cheaper roof, including:

48 lb. roofing nails @ 4 ¢
32 lb. roofing caps @ $4\frac{1}{2}$ ¢
1 bbl. roofing cement, 50 gal., @ 13 ¢
18 gal. roofing cement @ 15 ¢
Labor, $11.20

9. Find the cost per square, to the nearest cent, the total cost being $56.56.

Papering and Carpeting

EXERCISES

324. 1. Albert wishes to paper and carpet his room. It is 6 yards long and $4\frac{1}{2}$ yards wide. How many yards is it around the room? how many half yards?

2. How many strips of wall paper 18 inches wide will be required to paper the walls, if 2 strips are deducted for each of 2 windows and 2 strips for a door?

3. A roll of wall paper is 8 yards in length, and a double roll 16 yards. Since the ceiling of the room is 9 feet high, and the strips of wall paper need extend only from the baseboard to a few inches above the bottom of the border, Albert finds that after making all due allowances for matching the pattern he can cut 3 strips from a roll. How many rolls should he purchase, or how many double rolls?

4. How many lineal yards of border should he purchase, no allowance being made for matching the pattern?

5. If the ceiling paper is put on the long way of the room, it will require 1 roll for each strip. How many rolls of ceiling paper would then be required?

6. If the ceiling paper is put on the short way of the room, 3 strips can be cut from a double roll. How many double rolls would then be required?

7. Draw a diagram of the floor of Albert's room.

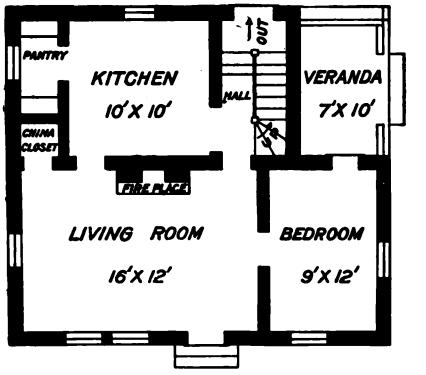
How many yards of carpet $\frac{3}{4}$ of a yard wide are required to cover the floor, no allowance being made for matching the pattern?

8. How many yards of matting 1 yard wide would be required to cover the floor?

9. How many square yards of linoleum $1\frac{1}{2}$ yards wide would be required? 2 yards wide?

WRITTEN EXERCISES

325. 1. Find the cost of papering the walls and ceiling of the bedroom on the first floor of this cottage, at 50¢ per roll for wall paper, 40¢ per roll for ceiling paper, and 30¢ per lineal yard for the border, supposing that 3 strips can be cut from each roll and that 2 strips may be deducted for each window and 3 strips for each door.



2. Find the cost of covering the bedroom floor with carpet 27 in. wide, at $1.40 per lineal yard.

3. Find the cost of covering the kitchen floor with linoleum, one strip 2 yd. wide and another $1\frac{1}{2}$ yd. wide, at $1.10 per square yard.

4. Find the cost of papering the living room, using the same kind of paper as for the bedroom, deducting 2 strips for each window, 3 strips for each door, and 3 strips for the fireplace.

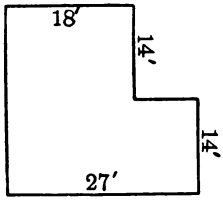
326. In practice it is not always worth while, or even possible, to compute the exact cost of papering or carpeting rooms. What is sought is the approximate cost, making allowances for doors and windows, for matching patterns, etc.

MISCELLANEOUS EXERCISES

327. 1. The ceilings of the house shown on the preceding page are 9 ft. 3 in. high. How much will it cost to have the bedroom plastered, down to the floor, at 30¢ per square yard, deducting 1 square yard for each window and 2 square yards for each door?

2. Find the cost of plastering the kitchen, making similar deductions.

3. Find the cost of plastering the living room, deducting for windows and doors, but not for the fireplace.



4. This is a diagram of the cellar. Find the cost of paving the cellar floor with concrete to a depth of 4 in., at $4.25 per cubic yard.

5. The cellar was dug 9 ft. deep. Find the cost of excavation at 45¢ per cubic yard.

6. Find the cost of 25 squares of red roofing slate at $8.50 per square.

7. During 6 months the output of a slate quarry in New York was 784 squares of red slate and 1515 squares of green slate. Find the value of the output at an average price of $8.75 per square for red slate and $7 per square for green slate.

8. In a city 32,772 lineal feet of cement walks were constructed in a year by the city at a cost of $26,575, and 14,265 lineal feet by property owners at a cost of $10,000. Find the average cost per lineal yard, to the nearest cent, of the cement walks constructed that year.

9. How much more per lineal yard did it cost to construct cement walks at public expense than at private expense?

10. The floor of a warehouse 120 feet by 48 feet could sustain a weight of 150 pounds per square foot. How many tons could the entire floor support?

11. In a recent year the United States produced 24,317,910 square yards of carpet $\frac{3}{4}$ of a yard wide. How many lineal yards of this width were produced?

12. The two sloping sides of a slate roof were each $30' \times 20'$. Find the total weight of the slate on the roof, if it weighed $6\frac{3}{4}$ pounds per square foot.

13. The lawn in front of Mr. Child's house was 45 ft. long and 30 ft. wide, with a path 3 ft. wide running the length of it. How many strips of sod 9 ft. long and 1 ft. wide were required to sod the lawn?

14. If 4000 strips of wood for golf sticks can be cut from $1333\frac{1}{3}$ feet of lumber, how many can be cut from 8000 board feet?

15. If one gallon of paint covers 650 square feet, how much paint is required to cover one side of a tight board fence 6 ft. 3 in. high and 78 ft. long?

16. A coal bin 12 ft. by 9 ft. and $5\frac{1}{3}$ ft. deep is filled with stove coal. How many tons of coal does it contain, if 1 ton occupies 36 cu. ft. of space?

17. Ice cut into blocks of a certain size, and well packed, is estimated to occupy 40 cu. ft. per ton. How many tons of ice can thus be packed into a space 24 ft. by 20 ft. by 18 ft.?

18. What is the cost of gravel per cubic yard, if \$21.50 is paid for 15 loads of $1\frac{2}{3}$ cubic yards each?

19. Find the total cost of 85 cubic yards of sand at 27¢ per cubic yard, to be carted 800 feet at an additional cost of 1¢ per cubic yard for each 50 feet of distance.

20. How many bricks are contained in a wall 6 feet high, 36 feet long, and 15 inches thick, if there are 600 bricks per cubic yard?

21. At $7.50 per cubic yard, find the cost of a concrete wall 39 feet long, 5 feet high, and 18 inches thick.

22. If the total weight of this wall when dry was 38,025 pounds, how much did it weigh per cubic foot?

23. The excavation for a large reservoir was let out to contractors in sections. One section, 135 ft. by 102 ft., was dug 6 ft. deep. Find the cost at $19\frac{1}{2}$¢ per cubic yard.

24. If it took 4 trains of 10 cars each to carry this material away, what was the average car load?

25. A dredge in Boston harbor excavated 6 cubic yards of mud every 40 seconds, emptying the mud into a scow. How much mud did it load on the scow in $2\frac{1}{2}$ hours?

26. How long should it take a steam shovel to dig a trench 10 ft. wide, 18 ft. deep, and $\frac{3}{4}$ of a mile long, if it can remove 1200 cu. yd. of earth per day?

27. A certain hotel courtyard 75 ft. by 50 ft. has a rubber pavement. How much did the pavement cost, at $32.70 per square yard?

28. What is the total weight of this rubber pavement, if it weighs $15\frac{1}{4}$ pounds for each square foot of surface?

29. Find the expense of paving $\frac{5}{8}$ of a mile of a street 45 feet wide with asphalt, at a cost of $1.96 per square yard.

30. How much did it cost to construct $4\frac{1}{2}$ miles of macadam road 14 feet wide, at 50¢ per square yard?

31. If 60 cubic feet of water are required for sprinkling 850 square yards of asphalt pavement, how many cubic feet of water would be used in sprinkling a strip of pavement 36 feet wide and 5100 feet long? how many gallons?

PERCENTAGE

328. **1.** A boy who had 60 marbles lost 10% of them. How many marbles did he lose?

In this problem what number is the base? the rate? What name is given to the result?

How do you find the percentage when the base and rate are given?

2. Elmer had 40 cents. If he spent 8 cents, what per cent of his money did he spend?

When the base and percentage are given, how is the rate found?

3. A newsboy sold $66\frac{2}{3}\%$ of his papers. If he sold 24 papers, how many did he have at first?

Tell how to find the base, when the rate and percentage are given.

EXERCISES

329. **1.** Put this diagram on the blackboard, and as the teacher points to each number outside of the square, tell rapidly 50% of it.

16	40	64
32	50%	48
88	24	72

2. Change the rate and find 25% of each number; then $12\frac{1}{2}\%$; 75%; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$.

Find quickly:

3. 20% of 5

4. 40% of 20

5. 60% of 45

6. 80% of 60

7. $33\frac{1}{3}\%$ of \$1.50

8. $16\frac{2}{3}\%$ of \$2.40

9. $66\frac{2}{3}\%$ of \$3.18

10. $83\frac{1}{3}\%$ of \$6.42

11. 5% of \$120

12. $\frac{2}{5}\%$ of \$250

13. 6% of \$700

14. 8% of \$500

15. What per cent of 48 is 24? 16? 12? 8? 6? 32? 36? 40? 18? 30? 42?

16. 9 is what per cent of 12? of 36? of 72? of 15? of 30?

17. 6 is what per cent of 18? of 20? of 36? of 48? of 60?

18. 12 is what per cent of 24? of 16? of 18? of 32? of 72?

What per cent of

19. \$64 is \$8? **22.** 90 rd. is 60 rd.? **25.** \$1.20 is \$.12?

20. \$80 is \$20? **23.** 45 lb. is 18 lb.? **26.** \$1.50 is \$.25?

21. \$42 is \$14? **24.** 56 bu. is 35 bu.? **27.** \$4.00 is \$.50?

Find the number of which 12 is

28. 50% **30.** $12\frac{1}{2}$% **32.** $37\frac{1}{2}$% **34.** 6% **36.** 12%

29. 20% **31.** $16\frac{2}{3}$% **33.** $66\frac{2}{3}$% **35.** 8% **37.** 24%

Find the number of which 15 is

38. $33\frac{1}{3}$% **39.** 40% **40.** $37\frac{1}{2}$% **41.** $62\frac{1}{2}$% **42.** $83\frac{1}{3}$%

43. 11 is 25% of what number?

44. 24 is 60% of what number?

45. 36 is 75% of what number?

46. 48 is 80% of what number?

47. 25 is $12\frac{1}{2}$% of what number?

48. 40 is $16\frac{2}{3}$% of what number?

49. 55 is $83\frac{1}{3}$% of what number?

50. 63 is $87\frac{1}{2}$% of what number?

51. How many inches is $16\frac{2}{3}$% of a foot?

52. During a certain week John was absent from school 2 half days. What per cent of the time was he in attendance?

53. A man spends \$80 annually for his clothes, and this is $12\frac{1}{2}$% of his income. What is his income?

54. A silver vase that weighs 30 ounces is 4% pure silver. What weight of pure silver does it contain?

55. Ethel failed on 3 of the 15 questions in her examination. What per cent of the questions did she answer correctly?

56. After traveling 120 miles Mr. Joy had completed 40 % of his journey. How long was his journey?

57. If a piece of brass is composed of 2 parts of copper to 1 part of zinc, what per cent of the brass is copper?

SUGGESTION. — What per cent of 1 + 2, or 3, is 2?

58. Gold coins of the United States contain 9 parts of pure gold to 1 part of alloy. What per cent is pure gold?

59. I rent 66⅔ % of my farm, or 220 acres. How many acres does the farm contain?

60. In a certain town 240 people, or 30 % of the inhabitants, are engaged in manufacturing. What is the population?

61. Of 450 crates of oranges shipped from Florida 9 were spoiled on the way. What per cent of the fruit was spoiled?

62. In a certain school 60 % of the pupils are girls. If there are 180 girls, how many pupils are there in the school?

63. Chickens lose 20 % of their weight in being dressed for market. A butcher buys 40 pounds of live chickens. How much will they weigh when dressed?

64. Mr. Mason bought 18 bushels of potatoes, and upon sorting them found that only 15 bushels were first class. What per cent of the potatoes were poor?

65. A grocer mixes 12 pounds of Java coffee with 36 pounds of Mocha. What per cent of the mixture is Java?

66. A local telephone company has 640 miles of line. If 12½ % of it is in a certain county, how many miles of the line are in that county?

67. If the 400 books on travel in a library are 5 % of all the books, how many volumes does the library contain?

68. If the cost of operating a railroad one year was $360,000, and 66⅔ % of this was expended by the freight department, what were the expenses of this department?

330. In expressions of rate the fractional part of 1 per cent is frequently written in the decimal form.

Thus, $18\frac{7}{20}\%$ may be written 18.35%, which means the same as .1835.

331. The rate is often given approximately to the nearest tenth or hundredth of 1 per cent, according to the degree of accuracy desired.

Thus, since $\frac{13}{14} = .928+$, $5\frac{13}{14}\%$ (to the nearest tenth of 1%) is 5.9% and (to the nearest hundredth of 1%) 5.93%.

The decimal equivalents of these expressions are .059 and .0593, respectively.

WRITTEN EXERCISES

332. Give the decimal equivalent of:

1. 34.8%	**3.** 67.23%	**5.** 7.6%	**7.** 8.27%
2. 22.6%	**4.** 40.91%	**6.** 5.4%	**8.** 9.99%

Express as per cent with the sign:

9. .257	**11.** .8724	**13.** .026	**15.** .0844
10. .421	**12.** .5063	**14.** .082	**16.** .0619

Express with the sign to the nearest *tenth* of 1 per cent; also to the nearest *hundredth:*

17. $6\frac{5}{33}\%$	**19.** $12\frac{16}{27}\%$	**21.** $86\frac{17}{36}\%$	**23.** $36\frac{19}{48}\%$
18. $4\frac{19}{21}\%$	**20.** $45\frac{29}{52}\%$	**22.** $24\frac{28}{79}\%$	**24.** $90\frac{61}{62}\%$

MISCELLANEOUS EXERCISES

333. Find to the nearest cent:

1. $6\frac{1}{4}\%$ of \$28.60	**7.** $16\frac{2}{3}\%$ of \$376.32
2. $8\frac{1}{3}\%$ of \$41.25	**8.** $37\frac{1}{2}\%$ of \$849.74
3. 23% of \$30.86	**9.** 26.3% of \$1260.80
4. 78% of \$54.75	**10.** 3.98% of \$4529.12
5. 4.7% of \$92.66	**11.** 78.24% of \$7246.45
6. 8.9% of \$75.90	**12.** 96.05% of \$8036.28

What per cent of

13. 297 is 99?
14. 8.25 is .66?
15. 544 is 3.4?
16. 1728 is 216?
17. 2240 is 784?
18. 4152 is 692?
19. \$35 is \$1.68?
20. \$48 is \$7.92?
21. \$75 is \$8.40?

Find the number of which

22. \$306.72 is 3.6%
23. \$465.60 is 75%
24. \$507.15 is 23%
25. \$812.16 is 4.7%
26. \$2698.29 is $87\frac{1}{2}$%
27. \$4295.25 is 24.9%
28. \$6384.90 is $83\frac{1}{3}$%
29. \$7844.62 is 28.63%

What per cent (to the nearest hundredth) of

30. 79 is 27?
31. 42 is 69?
32. 264 is 45?
33. 8.76 is 24?
34. 46.49 is 28.6?
35. 629.4 is 397?

36. A sample of coal was found to be 73.19% carbon. How much carbon is there in a long ton of such coal?

37. If coal yields 65.9% of its weight in coke, how many pounds of coal will yield 5931 pounds of coke?

38. One year a National League baseball team won 106 of the 153 games played. What per cent of the games did the team win?

Give baseball per cents, or "averages," to the nearest tenth of 1%.

39. A player who led his team in batting came to bat 490 times during the season and made 171 hits. What was his batting average?

40. Find the fielding average of a player who accepted 899 chances and made 15 errors in one season.

41. On an English highway 6000 vehicles passed a certain point in 12 hours. $3\frac{2}{5}$% of them were drawn by horses, $7\frac{1}{2}$% were electric street cars, 9.9% motor vehicles, and $79\frac{1}{5}$% bicycles. How many vehicles of each kind were there?

42. In a year when there were 994,762 pensioners in the United States 274,447 were widows of soldiers. Find, to the nearest tenth, the per cent of pensioners that were widows.

43. If 22.9% of the cost of operating a mine in Alaska was for labor that cost \$6434.90, find the cost of operating the mine.

44. One year there were 270,000 Indians in the United States, of whom 50,000 could read. Find, to the nearest tenth, the per cent of the Indian population that could read.

45. Find the number of bushels of wheat necessary to make 225 barrels of flour, if wheat yields 75% of its weight in flour.

46. In the manufacture of 432 lb. of plate glass, 151.2 lb. of sand, 146.88 lb. of limestone, 43.2 lb. of soda, 36.72 lb. of broken glass, and 54 lb. of other materials were used. What per cent of the whole was each substance?

47. Mr. Williams's income from his business one year was \$2125, which was 25% of the value of his stock. The next year a fire damaged his stock to the extent of \$1275. What per cent of the value of his stock did he lose by fire?

48. One year cotton goods to the value of \$4,904,204 were imported into the Philippine Islands. 45.05% came from Great Britain, 6.52% from the United States, and 15.48% from Spain. What was the value of the cotton goods imported from each country?

49. The following table shows the production of certain furs for one year:

Kind of Fur	World	United States
Otter	33,640 skins	14,600 skins
Beaver	66,000 "	8,000 "
Fur seal	95,485 "	24,000 "
Mink	728,000 "	578,000 "
Muskrat	5,285,000 "	4,035,000 "

Find, to the nearest tenth for each kind of fur, the per cent produced by the United States.

334. Sum or difference of a number and some per cent of it.

1. How many are 6 increased by a number equal to $\frac{1}{3}$ of 6?
   6 plus $\frac{1}{3}$ of 6 = ? $\frac{4}{3}$ of 6 = ?
2. How many are 6 decreased by a number equal to $\frac{1}{3}$ of 6?
   6 less $\frac{1}{3}$ of 6 = ? $\frac{2}{3}$ of 6 = ?
3. How many are 12 increased by 25 % of itself?
   100 % + 25 %, or 125 %, of 12 = ?
4. How many are 12 decreased by 25 % of itself?
   100 % − 25 %, or 75 %, of 12 = ?
5. Find 15 increased by 20 %; decreased by $33\frac{1}{3}$ %.

EXERCISES

335. 1. The population of a certain town three years ago was 250. If it has increased 20 %, what is the population now?

2. One week Frank made $3 selling papers. How much did he make the second week if his profits decreased 25 %?

3. A train that was running 50 miles an hour increased its speed 4 %. How many miles an hour was it then running?

4. Mr. Thompson's flock of 240 sheep was decreased $12\frac{1}{2}$ % by an epidemic. How many sheep had he after the epidemic?

5. A canning factory using 6600 bu. tomatoes a season increased its capacity $16\frac{2}{3}$ %. How many bushels did it then use?

6. Formerly there were 400 fishermen in Digby, but their number has decreased 5 %. How many are there now?

7. During a certain month 200 girls were employed in a factory, and the next month 210. Find the per cent of increase.

SUGGESTION. — What per cent of 200 is 10?

8. One month Mr. Wilson burned 3000 ft. of gas in his house, and the next month 2400 ft. What was the per cent of decrease?

WRITTEN EXERCISES

336. 1. Mr. Charles invested $6540 in business, and a year later increased his capital $16\frac{2}{3}\%$. How much was his capital then?

2. Two years ago a lawyer's income was $4868, but last year it decreased $12\frac{1}{2}\%$. What was his income last year?

3. Flour that sold for $4.85 was advanced in price to $5.82. Find the per cent of advance.

4. A mason's helper working for $1.75 per day had his pay increased $14\frac{2}{7}\%$. How much did he then receive?

5. When the number of representatives in Congress from the state of New York was increased from 34 to 37, what was the per cent of increase, to the nearest tenth?

6. The income from the fisheries of Virginia one year was $3,179,498. In three years it had increased 45.09%. What was the income for the third year?

7. During ten years the number of males employed in the manufacture of gloves in the United States increased from 2741 to 4503, and of females from 3675 to 7768. Find, to the nearest tenth, the per cent of increase of each.

8. The unwashed wool from a flock of sheep one year weighed 8840 lb. and was sold for $2210. The next year the yield, 9282 lb., was sold for $1949.22. Find the increase per cent in the yield and the decrease per cent in the price per pound.

STATE OF NEVADA

Year	Population
1860	6,857
1870	42,491
1880	62,266
1890	45,761
1900	42,335

9. Find, to the nearest tenth, the per cent of increase in population in Nevada from 1860 to 1870; from 1870 to 1880.

10. What was the per cent of decrease from 1880 to 1890? from 1890 to 1900?

11. Find, to the nearest hundredth, the per cent of decrease from 1870 to 1900.

337. Finding a number when the number increased by some per cent of itself is given.

1. If 3 times a number is 15, how do we find the number?

2. If $1\frac{1}{3}$ times a number is 8, what is the number? A number increased by $\frac{1}{3}$ of itself equals 8. What is the number?

3. If 1.25, or $1\frac{1}{4}$, times a number is 10, what is the number? A number increased by .25 of itself equals 10. What is the number?

4. If 100% of a number is 12, what is the number? If 120% of a number is 12, what is the number? A number increased by 20% of itself equals 12. What is the number?

Find the number of which

5. 11 is 110%

6. 24 is 120%

7. 25 is 125%

8. 32 is 200%

9. 45 is 150%

10. 26 is 130%

What number increased by

11. 25% of itself equals 250?

12. 20% of itself equals 360?

13. $33\frac{1}{3}$% of itself equals 120?

14. $12\frac{1}{2}$% of itself equals 108?

WRITTEN EXERCISES

338. 1. What number increased by 32% of itself equals 165?

SOLUTION. — The number + 32% of it = 132% of it = 165; that is, 1.32 times the number = 165.

Hence, the number = 165 ÷ 1.32 = 125.

2. What number increased by 75% of itself equals 672?

SUGGESTION. — Since 75% of the number is $\frac{3}{4}$ of it, 672 is $1\frac{3}{4}$ times the number.

What sum of money increased by

3. 43% of itself = \$354.64?

4. 68% of itself = \$890.40?

5. $16\frac{2}{3}$% of itself = \$693.84?

6. $62\frac{1}{2}$% of itself = \$459.16?

7. A clerk's salary was increased 18%. He then received $1652 a year. What was his salary before the increase?

8. A typewriter can transcribe her notes at the rate of 50 words per minute, which is 25 % faster than she could do it six months ago. What was her rate then?

9. The number of children in an orphan asylum has increased 8% this year, and there are 486 children in it at present. How many children were there in the asylum last year?

10. I paid $1250 for my automobile, or 60% more than for my horse and carriage. How much did my horse and carriage cost?

11. The cost of living for a family one year was $442, which was an increase of 4% over the cost for the year before. Find the cost for the year before.

12. The circulation of a publication has increased 26% in three years, and is at present 177,030. What was the circulation three years ago?

13. During about 40 years from the close of the Civil War, the railway system of the United States increased 500%. If it comprised 210,000 miles at the end of this period, what was the mileage at the close of the war?

14. The production of woolen cloth in the United States increased 323% in six years, and then amounted to 274,950 yards per week. How many yards per week were produced at the beginning of the time?

15. A gas meter that ran 2.5% too fast registered the passage of 65,600 cubic feet of gas in a year. What was the actual amount of gas that passed through the meter?

16. The yearly value of the meat packing and slaughtering products in Kansas City increased 84.8% during ten years, and at the end of that time was $33,859,980. What was the value of these products at the beginning of the decade?

339. Finding a number when the number decreased by some per cent of itself is given.

1. What part of a number is left when $\frac{1}{3}$ of it has been taken away?

If $\frac{2}{3}$ of a number is 6, what is the number?

A number decreased by $\frac{1}{3}$ of itself equals 6. What is the number?

2. What part of a number is left when .25 of it has been taken away?

If .75, or $\frac{3}{4}$, of a number is 9, what is the number?

A number decreased by .25 of itself equals 9. What is the number?

3. What per cent of a number is left after taking away 10% of it? $100\% - 10\% = ?$

If 90% of a number equals 18, what is the number?

A number decreased by 10% of itself equals 18. What is the number?

4. What per cent of anything is left after taking away $12\frac{1}{2}\%$ of it? $100\% - 12\frac{1}{2}\% = ?$

If $87\frac{1}{2}\%$, or $\frac{7}{8}$, of a number is 21, what is the number?

A number decreased by $12\frac{1}{2}\%$ of itself equals 21. What is the number?

What number decreased by

5. 15% of itself equals 85?
6. 20% of itself equals 24?
7. $66\frac{2}{3}\%$ of itself equals 15?
8. $87\frac{1}{2}\%$ of itself equals 11?

WRITTEN EXERCISES

340. 1. What number decreased by 36% of itself equals 208?

SOLUTION. — The number − 36% of it = 64% of it = 208; that is, .64 of the number = 208. Hence, the number = 208 ÷ .64 = 325.

2. What number decreased by $16\frac{2}{3}\%$ of itself equals 485?

SUGGESTION. — Since $16\frac{2}{3}\%$ of the number is $\frac{1}{6}$ of it, 485 is $\frac{5}{6}$ of the number.

What number decreased by

3. 32 % of itself = $166.60? 5. 37½ % of itself = $453.75?

4. 65 % of itself = $332.50? 6. 83⅓ % of itself = $742.36?

7. Walter lost 5 % in weight one year and then weighed 114 pounds. How much did he weigh at the beginning of the year?

8. If beef loses 20 % of its weight by roasting, find the weight, before cooking, of 14 pounds of roast beef.

9. Mr. Ford had 76 % of his money invested in houses, and the rest, $7680, he put in the bank. How much money had he?

10. A man who had property worth $15,600 found that this was 60 % less than he owed. How much did he owe?

11. If cloth shrinks 5½ % of its length in being sponged, find the original length of a sponged piece 37.8 yards long.

12. Coal sold at $5.50 per ton one winter, or 8⅓ % less than the price the winter before. Find the price the winter before.

13. If 6½ % of those engaged in a battle were killed and 39,270 survived, how many were engaged in the battle?

14. Some flour was damaged 25 % in transportation. If 600 barrels arrived in good condition, how many were shipped?

15. One year the income of the railroads of the United States averaged $11.89 for every dollar they spent for fuel. Find the average three years before, which was 30 % less.

16. In a state in which 19.8 % of the population are negroes, there are 952,375 whites. Find the population of the state.

17. A merchant had a stock of hats that cost $2.50 each. When he had sold 75 % of them, the cost value of the rest was $360. How many of these hats did he have in stock at first?

18. If a city decreased in population 15 % each year for two successive years and then had 8750 inhabitants, what was the population before the decrease?

Profit and Loss

341. **1.** A jeweler bought a watch for $80 and sold it at a gain of 25 % of the cost. How much did he gain?

For how much did he sell the watch?

2. Mr. Rich paid $1200 for an automobile and after using it a year he sold it at a loss of $33\frac{1}{3}$ % of the cost. How much did he lose? How much did he receive for it?

3. Sugar that cost 5 cents a pound was sold for $5\frac{1}{2}$ cents a pound. How much was gained per pound? What part of the cost was the gain? What per cent of the cost was the gain?

4. When the gain equals $\frac{1}{5}$ of the cost, what per cent is gained? What per cent of the cost is the selling price?

5. When $\frac{1}{4}$ of the cost is lost, how is the loss expressed in per cent? How is the selling price expressed in per cent?

6. A dealer sold a gasoline launch for $500 and gained 25 % of the cost. How much did the launch cost?

7. A bicycle was sold for $40 at a loss of 20 %. Find the cost.

342. The per cent of gain or of loss is reckoned on the *cost* or the *sum invested.*

EXERCISES

343. **1.** A suit of clothes that cost $20 was sold at a profit of 25 %. How much was gained?

2. Gloves that were bought for 60¢ per pair were retailed at a gain of $33\frac{1}{3}$ %. What was the selling price?

3. A grocer made 10¢ by selling a pound of tea for 40¢. What per cent of the cost did he gain?

4. A city lot that cost $500 was sold at a loss of 10 %. How much was lost?

5. A merchant bought silk for \$1 per yard and sold it at a profit of 25 %. What was the selling price?

6. Coffee bought for 28¢ a pound was sold for 35¢ a pound. What was the gain per cent?

7. When a hat that cost \$1.50 was sold at an advance of 50 %, what was the selling price?

8. A merchant sold a rug for \$48, thus gaining \$8. What was the gain per cent?

9. Some goods were sold at a profit of \$50. If 10 % was gained, find the cost of the goods.

10. A horse bought for \$150 was sold for \$250. What was the gain per cent?

11. A carriage was bought for \$120 and sold for \$100. Find the loss per cent.

12. Find the selling price of cloth bought at 75¢ a yard and sold at a profit of 20 %.

13. By selling coal at \$6 a ton a gain of 20 % was made. How much did the coal cost?

14. A clothier sold 5 overcoats for what 8 cost him. What was the gain per cent?

15. A dry goods merchant sold flannel for 36¢ per yard at a gain of $12\frac{1}{2}$ %. How much did the flannel cost him?

16. Some goods that cost \$3000 were damaged by fire and sold at a loss of 12 %. How much was lost?

17. A man bought a house for \$4000 and later sold it at a gain of 15 %. What was the selling price?

18. A furniture dealer sold a parlor suit at a gain of $16\frac{2}{3}$ %. If he sold it for \$210, how much did he pay for it?

19. What was the loss per cent on a farm bought for \$3200 and sold for \$2400?

WRITTEN EXERCISES

344. **1.** Find the gain on a stove that cost \$36 and was sold for 15 % more than it cost.

2. I bought a phonograph for \$25 and sold it at a loss of 34 %. How much did I lose?

3. When raw silk costs \$3.84 per pound, for how much per pound must it be sold to make a profit of $16\frac{2}{3}$ %?

4. Grain that cost 66¢ per bushel was damaged so that it was sold for $41\frac{1}{4}$¢ per bushel. What was the per cent of loss?

5. A grocer paid \$14.40 for a tub of butter weighing 60 pounds. Find his selling price per pound, if he gained $8\frac{1}{3}$ %.

6. A quantity of leather that cost \$1648 was sold at a gain of $12\frac{1}{2}$ %. For how much was it sold?

7. A stationer bought blank note books at \$1.08 per dozen and sold them at 15¢ each. What per cent did he gain?

8. A haberdasher sold shirts at \$1.50 each, thereby gaining 20 %. How much did he pay for them per dozen?

9. Find the gain per cent on a gross of brooms bought for \$34.56 and sold at 28¢ each.

10. Find the cost per dozen of infants' woolen hose that retail for 25¢ a pair at a gain of 60 %.

11. A man who paid \$8250 for a yacht sold it at a loss of 14 %. What was the selling price?

12. A lumber merchant sold 4800 feet of lumber for \$103.68, gaining 8 %. How much did it cost him per thousand feet?

13. Find the gain to New York fruit shippers, on a week's shipment of California oranges, consisting of 117 car loads of 362 boxes each, if they cost \$1.35 per box in New York and are sold at auction at an average profit of 20 %.

14. If apples are purchased for 60¢ per bushel and sold by the quart at a gain of $33\frac{1}{3}$ %, find the selling price per quart.

15. Codfish bought for $3.50 per hundredweight was sold in 5-pound boxes at 21¢ a box. Find the gain per cent.

16. A grocer bought a quantity of sugar at $4.87½ per hundredweight and sold it at a profit of 14⅔ %. What was his selling price per pound?

17. What per cent is gained by buying pork at $22.50 per barrel (200 lb.) and retailing it at 14¢ a pound?

18. Find the gain per cent, to the nearest tenth, on neckties bought at $1.32 per dozen and sold at 25¢ each.

19. A man bought a ring for $35, which was 12½ % less than its value. He sold it for 12½ % more than its value. Find his gain.

20. When Georgia peaches are bought at $2.25 per bushel crate and are sold at 12¢ a quart, what is the gain per cent, supposing that each crate yields only 30 quarts of salable fruit?

21. Shoes that cost $18.48 per dozen pairs were sold at $2 per pair. Find the per cent of gain to the nearest tenth.

22. Mr. Jordan bought two houses, one for $2400 and the other for $5200. He sold the first at 120 % of the cost, and on the second he lost 4 %. Find his net gain on both transactions.

23. A dealer gained 20 % on flour that he sold for $6 a barrel. The market advanced, and he sold the rest of his stock for $6.50 a barrel. What was his per cent of gain after the advance?

24. A merchant's complete stock of goods cost $50,000. He sold it at an average advance of 15 % above cost and lost 2¼ % of his sales in bad debts. Find his gain.

25. A wholesale fruit dealer paid $350 for oranges, $280 for bananas, and $320 for peaches. He sold the oranges at cost, the bananas at 130 % of the cost, and the peaches at a loss of 5 %. Did he gain or lose on all, and how much?

Commission

345. **1.** I employed a man called a *commission merchant* to sell my potatoes in the city. He sold the crop for $800, and charged me 5% of this sum for his services. How much did he charge for his services; that is, what was his *commission?*

He took out the commission, $7 for carting the potatoes, and $3 for storing them, and then sent me the rest of the $800. How much did he send me; that is, what were the *net proceeds?*

2. An *agent* bought some cloth for $400, charging 3% of the cost for his services. How much was his commission?

3. A lawyer collected a debt of $500, receiving 10% of it for his services. What was his commission?

346. A person who buys or sells goods or transacts business for another is called an **agent.**

347. The compensation of an agent is usually reckoned as some per cent of the *value* involved, and is called **commission.**

Thus, a seller's commission is some per cent of the amount of sales; a buyer's commission is some per cent of the cost; a collector's commission is some per cent of the money collected.

348. The sum left after the commission and other expenses have been paid is called the **net proceeds.**

EXERCISES

349. **1.** An agent received 20% commission for selling pictures. How much did he earn by selling 50 at $2 each?

2. How much was a collector's commission on $1000 collected, if he charged 5%?

3. A book agent's sales one month amounted to $200. How much did he make that month, if his commission was 40%?

4. A commission merchant sold 100 barrels of flour at $6 a barrel. Find his commission at 3%.

5. The value of the maps sold by an agent one summer was $650. Find his commission at 50%.

6. Find the commission at 2% on 400 tons of coal sold for $5 a ton.

7. A lawyer received $15 for collecting a debt of $300. What was his rate of commission?

8. How much did a commission merchant receive for selling 50 barrels of apples at $2 per barrel, if the rate of commission was 7%?

9. How much was the builder's commission at 10% on a house that cost $4500?

Find the architect's commission at 5%.

WRITTEN EXERCISES

350. 1. A commission merchant sold 2000 bushels of corn at 56¢ a bushel, commission 2%. He paid $123.30 freight and $27 cartage. Find the commission and the net proceeds.

SOLUTION

Amount of sales = 2000 × 56¢ =		$1120.00
Commission = 2% of $1120 =	$ 22.40	
Freight and cartage . .	150.30	172.70
Net proceeds		$ 947.30

2. Find the commission and net proceeds of a sale of 150 dozen bunches of celery at 36¢ a dozen, if the rate of commission was 5%.

3. My agent sold goods for me to the amount of $4620. If he paid $85 for cartage and other expenses and charged 3% commission, what were the net proceeds?

4. An agent collected $745 for his employer and charged 7% commission. Find the commission and the net proceeds.

5. A commission merchant purchased rice for me to the amount of $2840.50. Find the commission at $3\frac{1}{2}$%.

6. Find the auctioneer's commission on a sale of rugs, the amount of the sale being $97,546.50, and the rate of commission 2%.

7. Find the net proceeds from the sale of 250 bags of coffee, averaging 132 pounds each, at 14½¢ per pound, reckoning the commission at 2%.

8. What is the weekly income of a clerk who receives $8 a week and a commission of 4 % of his sales, if his sales average $250 a week?

9. An agent sold 4 sewing machines at $35 each, receiving a commission of 40 %. The expense of delivering them was $2.25 each. Find the commission and the net proceeds.

10. Find the commission and the net proceeds on a sale of 225 bales of cotton, averaging 500 pounds each, at 11¢ per pound. The rate of commission was 3%. Freight and other expenses amounted to $125.

Find the commission and the net proceeds:

	Produce	Quantity	Price	Rate of Commission
11.	Cabbages	500 bbl.	75¢	5 %
12.	Onions	250 bu.	85¢	6 %
13.	Apples	360 bbl.	$1.75	10 %
14.	Peaches	120 baskets	$1.25	8 %
15.	Blackberries	960 qt.	12¢	7½ %
16.	Butter	2880 lb.	18¾¢	5 %
17.	Cheese	1280 lb.	8¼¢	5 %
18.	Eggs	2250 doz.	22¢	5 %
19.	Chickens (live)	1760 lb.	13¢	10 %
20.	Chickens (dressed)	2450 lb.	16¢	5 %

21. A lawyer collected 80% of a debt of $2500 and charged 10% commission. How much did the creditor receive?

22. What was the income last year of a commercial traveler whose sales amounted to \$105,620, if his commission was $2\frac{3}{4}\%$?

23. How much business must a commission merchant do to earn \$125, if his average commission is $2\frac{1}{2}\%$?

24. Find the rate of commission, if the net proceeds from a sale of \$5000 are \$4875.

25. The net proceeds of a sale of produce were \$2451. If the commission was \$129, find the rate of commission.

26. If a lumber agent sold 80,000 feet of lumber at \$24 per thousand and received a commission of $4\frac{1}{4}\%$, how much was his commission?

27. An agent bought a block of 14 houses, each worth \$3200, on a commission of $1\frac{1}{2}\%$. How much did he receive for his services?

28. The rent of a house was $\$37\frac{1}{2}$ per month, and the agent retained out of that sum a commission of $\$1\frac{1}{2}$ per month. What was the rate of commission?

29. Find the architect's commission on an office building that cost \$24,000, if he received $1\frac{3}{4}\%$ for drawing up the plans, and $3\frac{1}{4}\%$ for superintending the construction.

30. A commercial traveler who sold goods on a commission had an income one year of \$3000 and sold goods amounting to \$120,000. What was his rate of commission?

31. The commission at $3\frac{1}{2}\%$ for selling a quantity of oats was \$189. How many bushels were sold, if the price received was 30¢ per bushel?

32. A Cincinnati packer sent 150 barrels of mess pork to a commission merchant in Syracuse, who sold it at \$12.30 a barrel, paid \$77 freight charges, 10¢ a barrel cartage, and 4¢ a barrel storage. Find the commission at $2\frac{3}{4}\%$ and the net proceeds.

Commercial Discount

351. To meet the varying conditions of the market, discounts are given from the prices published in catalogues and price lists, the list prices usually being higher than market prices.

Sometimes, when goods are sold to be paid for at a future time, a discount is allowed for payment before that time.

It often happens that several successive discounts are allowed. The first is a discount from the list price, the second from the remainder, the third from the second remainder, and so on.

WRITTEN EXERCISES

352. **1.** Find the net price of an article listed at \$17.40, if the discounts are 25% and 20 %.

List price	\$17.40
Less 25 %	4.35
Remainder	13.05
Less 20 %	2.61
Net price	\$10.44

The first discount is 25% of \$17.40, or \$4.35. Subtracting, we find the remainder, \$13.05. The second discount is 20% of \$13.05, or \$2.61. Subtracting, the net price is found to be \$10.44.

NOTE. — Find the discounts in the order (20 %, 25 %) and thus show that the result is the same in whatever order they are taken.

Find the net price of articles listed and discounted as follows:

2. \$25; 20 %, 10 %.

3. \$40; 5 %, 15 %.

4. \$3.75; $33\frac{1}{3}$ %, $12\frac{1}{2}$ %.

5. \$5.70; 40 %, $16\frac{2}{3}$ %.

Find the net price, net cost, and total discount on each:

	ARTICLES	PRICE	RATES OF DISCOUNT
6.	3 doz. food choppers	\$24.00 per doz.	30 %, 10 %
7.	$4\frac{1}{2}$ doz. meat cleavers	9.00 per doz.	$33\frac{1}{3}$ %, 5 %
8.	6 doz. butcher knives	6.50 per doz.	40 %, 10 %
9.	$5\frac{1}{2}$ doz. coffee mills	8.00 per doz.	35 %, 20 %
10.	15 lawn mowers	12.00 each	60 %, 10 %, 5 %

11. A music dealer sold a piano listed at $300 for 40% and 10% off. How much did he receive?

12. Find the net cost to a retailer of $2\frac{1}{2}$ dozen hammocks, the list price of which is $16 per dozen; discounts 20%, $12\frac{1}{2}$%.

13. The list price of a set of George Eliot's works is $30. If 40% discount is given, and 10% special, find the net cost.

14. Ford & Bond sold me $4\frac{1}{2}$ dozen thermometers at $15 per dozen; discounts 20%, $7\frac{1}{2}$%. Find the total discount.

15. When the list price of sash cord is 30¢ per pound, with discounts of 15% and 5%, how much will 200 pounds cost?

16. George Snyder & Co. bought 60 bathrobes at discounts of 25% and 20%. If the bathrobes were listed at $6 each, what was the net cost?

17. How much must be paid for 6 sets of harness, the list price being $15.20 each, if there are discounts of 25% and 5%?

18. How much did I pay for 75,000 shipping tags listed at 25¢ per thousand; discounts 25%, 5%?

19. In paying for 2 dozen beaver traps, listed at $16.50 per dozen, a sporting goods dealer received discounts of 60% and 10%. How much did he pay for the traps?

20. A dealer bought 12 six-pound sledges, listed at 30¢ per pound, at discounts of 75% and 10%. Find the net cost.

21. Ryder & Co. bought 720 pounds of galvanized sheet iron at discounts of 70% and $2\frac{1}{2}$%. If the list price was 15¢ per pound, how much was the net cost?

22. Mr. Fuller bought 80 feet of steel fence at $1.10 per foot, and 2 gates at $3.50 each. If he received a discount of 15%, and 5% for cash, what was the net cost?

23. I bought 2500 feet of fuse, the list price of which was $3.20 per 1000 feet, and received a discount of 10% from the list price and $2\frac{1}{2}$% for paying cash. How much did it cost me?

353. Discounting bills.

When there are discounts that are the same for all items of a bill, it is customary to find the amount of the bill first and then the discount on the amount.

354. The amount of a bill less all discounts is called the **net amount.**

The amount of a bill before it is discounted is sometimes called the **gross amount.**

WRITTEN EXERCISES

355. In each of the following, find the net amount from the given gross amount and discount:

1. \$200; 20%, 5%
2. \$400; 25%, 10%
3. \$600; 10%, 10%
4. \$300; 15%, 20%
5. \$720; 50%, 10%
6. \$850; 12%, 18%
7. \$230; 5%, 2%
8. \$496; $16\frac{2}{3}$%, 4%
9. \$560; $37\frac{1}{2}$%, 15%
10. \$744; $12\frac{1}{2}$%, $33\frac{1}{3}$%
11. \$1000; 10%, 10%, 5%
12. \$2400; 25%, 20%, 8%

13. What is the difference between a discount of 35% on a bill of \$320, and two successive discounts of 20% and 15%?

14. Shafer & Son received discounts of 40% and 10% on a bill of \$350 for silver plate. Find the total discount.

15. A bill for school supplies amounted to \$475.50. What was the cash payment if there was a discount of $33\frac{1}{3}$% and 5% for cash?

16. From a bill of \$840 for window glass, 90% and 10% discounts were allowed. What was the net amount of the bill?

17. Find the total discount on a bill of goods amounting to \$1280, with discounts of $12\frac{1}{2}$% and of 30%.

18. What is the net amount of a bill of \$496 for tinware, if the discounts are 75% and 10%, and 5% for cash?

19. The *terms* of the following bill are a trade discount of 5 % and an additional discount of 2 % for "cash."

Verify the gross amount and the net amount, if cash is paid.

Boston, Mass., April 7, 1906

Mr. Ralph Boyd,
72 Tremont St.

Bought of A. G. CROOKS & CO., 284 FEDERAL ST.
WHOLESALE GROCERS

TERMS: Trade 5%, 2% cash.

10 pails ginger,	250 lb.	23¢	$57	50		
15 boxes cloves,	150 lb.	15¢	22	50		
20 " cinnamon,	200 lb.	27¢	54			
			134			
Discounts 5%, 2%			9	25	124	75

Received payment,
A. G. Crooks & Co.

Make out the bills and find the net amount of each:

20. T. W. Cook bought of Jas. Monroe: 8 grindstones @ $5.50; 15 oil stoves @ 60¢; 1⅔ doz. scythes @ $6. Discounts, 33⅓ %, 10 %.

21. William Cline bought of H. N. Burt: 6 doz. sets laundry irons @ $1.50; 36 tea kettles @ 65¢; 60 coffee pots @ 45¢; 3 gross egg beaters @ $10.20. Discounts 40 %, 15 %.

22. Joseph Child bought of Walter White: 3 doz. hammers @ $6; 2½ doz. screwdrivers @ $7; 1¾ doz. saws @ $16; 15 bench planes @ $1.90. Discounts 25 %, 10 %.

23. A. L. Dewitt bought of Ward, Craft & Co.: 30 linen tablecloths @ $3.50; 40 doz. napkins @ $3; 60 doz. handkerchiefs @ $2.25. Discounts 15 %, 5 %.

Marking Goods

356. Merchants mark their goods to show the *cost* and the *selling price*, usually writing the first above a horizontal line and the second below.

To prevent customers from reading the mark and discovering the gain, some private mark is usually adopted.

The most usual device employed is some word or words of ten different letters to represent the ten Arabic numerals.

This is called a **key**.

Thus, if the key is "White Sambo," the corresponding letters and figures are

w	h	i	t	e	s	a	m	b	o
1	2	3	4	5	6	7	8	9	0

357. To avoid repeating a letter and thus giving a clew to the key, an extra letter, called a **repeater**, is used.

Thus, if the cost of an article is \$1.75 and the selling price \$2.99, using the key "White Sambo" with *k* for the repeater, the mark would be $\frac{\text{w.ae}}{\text{h.bk}}$.

In the exercises, use this key and repeater unless otherwise specified.

WRITTEN EXERCISES

358. Interpret the following and find the per cent of gain for each:

1.	$\frac{\text{w.he}}{\text{w.eo}}$	**3.**	$\frac{\text{.tm}}{\text{.et}}$	**5.**	$\frac{\text{st}}{\text{mk}}$	**7.**	$\frac{\text{h.ke}}{\text{h.mk}}$	**9.**	$\frac{\text{ia.eo}}{\text{ti.he}}$
2.	$\frac{\text{h.to}}{\text{i.ho}}$	**4.**	$\frac{\text{.ae}}{\text{w.ho}}$	**6.**	$\frac{\text{is}}{\text{th}}$	**8.**	$\frac{\text{i.to}}{\text{t.he}}$	**10.**	$\frac{\text{et.ae}}{\text{sa.mb}}$

In the following fill in the selling price at $33\frac{1}{3}\%$ above cost:

11.	$\frac{\text{.ht}}{\cdots}$	**13.**	$\frac{\text{t.ih}}{\cdots}$	**15.**	$\frac{\text{w.tk}}{\cdots}$	**17.**	$\frac{\text{bs}}{\cdots}$	**19.**	$\frac{\text{hm.ks}}{\cdots}$
12.	$\frac{\text{.sb}}{\cdots}$	**14.**	$\frac{\text{s.bo}}{\cdots}$	**16.**	$\frac{\text{a.mb}}{\cdots}$	**18.**	$\frac{\text{wih}}{\cdots}$	**20.**	$\frac{\text{ts.ie}}{\cdots}$

Fill in the cost mark in each of the following, the given selling price being 20% above cost:

21. $\frac{\cdots}{.tm}$	**23.** $\frac{\cdots}{i.ok}$	**25.** $\frac{\cdots}{te}$	**27.** $\frac{\cdots}{i.th}$	**29.** $\frac{\cdots}{ik}$
22. $\frac{\cdots}{.sk}$	**24.** $\frac{\cdots}{a.eo}$	**26.** $\frac{\cdots}{ma}$	**28.** $\frac{\cdots}{t.bm}$	**30.** $\frac{\cdots}{sm.ts}$

Mark articles to sell at $37\frac{1}{2}\%$ above the following costs:

31. 24¢	**33.** 88¢	**35.** \$2.80	**37.** \$4.48	**39.** \$176
32. 40¢	**34.** 96¢	**36.** \$7.20	**38.** \$6.80	**40.** \$224

41. How shall a book dealer mark a book that cost \$1.20 so that he may give a discount of 16% from the marked price and still make a profit of 40%?

SOLUTION. 140% of \$1.20 = \$1.68, selling price.

Since the selling price is 16% less than the marked price, \$1.68 is 84% of the marked price.

Then the marked price = \$1.68 ÷ .84 = \$2.

Hence the complete mark should be $\frac{w.ho}{h.ok}$.

Mark articles that cost as follows so that there will be a profit of 25% after giving the indicated discount:

42. 32¢, discount 20%	**45.** \$3.40, discount 15%
43. 72¢, discount 10%	**46.** \$5.25, discount $12\frac{1}{2}\%$
44. \$48, discount 25%	**47.** \$6.38, discount $16\frac{2}{3}\%$

Find the cost of one article when billed by the dozen as follows, and give complete marks (profit 40%), using the key "birthplace," repeater n, for the cost, and "White Sambo," repeater x, for the selling price:

48. Hats, \$21	**52.** Shirts, \$18
49. Caps, \$15.60	**53.** Gloves, \$16.80
50. Scarfs, \$24	**54.** Shoes, \$30
51. Neckties, \$7.20	**55.** Rubbers, \$8.40

INTEREST

359. 1. I hired a bicycle at 20 ¢ an hour. If I returned it at the end of 2 hours, how much did I pay for its use?

2. I rented Mr. Ward's house at $25 a month. How much did I pay for the use of the house for a year?

If this house was worth $3000, what per cent of its value did I pay for using it a year?

If instead of using Mr. Ward's house I had used $3000 of his money for a year, should I not have paid him for the use of the money?

3. I used $400 of another man's money for one year. When I returned the money, I also paid him a sum equal to 6 % of it.

How much did I pay for the use of $400 for a year, that is, how much *interest* did I pay?

At 5 % a year how much is the interest on $200 for 1 year? for 2 years? for 3 years?

4. I borrowed $500 for 1 year at 6 % interest. Besides paying back the *principal* sum, $500, how much interest did I pay?

How much in all did I pay at the end of the year, that is, what was the *amount?*

Find the amount of $100 at 7 % interest for 1 year; for 2 years; for 4 years; for 7 years.

5. Find the amount of $500 at 8 % interest for 1 yr.; for 5 yr.

360. The sum paid for the use of money is called **interest.**

361. The money for the use of which interest is paid is called the **principal.**

362. The *per cent* of the principal paid each year for interest is called the **rate.**

363. The sum of the principal and the interest is called the **amount.**

EXERCISES

364. Find the interest for 1 year on :

1. \$100 at 8%
2. \$300 at 6%
3. \$800 at 4%
4. \$600 at 5%
5. \$40 at 4%
6. \$60 at 5%
7. \$50 at 3%
8. \$80 at 2%
9. \$150 at 3%
10. \$900 at 7%
11. \$1000 at 6%
12. \$2500 at 2%

Find the interest at 5% on :

13. \$100 for 1 yr.; for 2 yr.
14. \$400 for 1 yr.; for 4 yr.
15. \$200 for 1 yr.; for 5 yr.
16. \$300 for 1 yr.; for 4 yr.
17. \$800 for 1 yr.; for 2 yr.
18. \$400 for 1 yr.; for 5 yr.

WRITTEN EXERCISES

365. 1. What is the interest on \$94.65 for 3 years at 6%?

SOLUTION

\$94.65, principal
.06, rate
\$5.6790, interest for 1 year
3
\$17.037, interest for 3 years

Hence the interest on \$94.65 for 3 years at 6% is \$17.04.

The interest equals the principal multiplied by the rate multiplied by the number expressing the time in years.

Find the interest on :

2. \$36.24 for 1 yr. at 5%
3. \$87.38 for 3 yr. at 4%
4. \$56.80 for 2 yr. at 7%
5. \$2865 for 4 yr. at 8%
6. \$7575 for 6 yr. at 3%
7. \$9830 for 3 yr. at 6%
8. \$125.50 for 2 yr. at 2%
9. \$5500 for 3 yr. at $3\frac{1}{2}$%
10. \$4248 for 5 yr. at $4\frac{1}{4}$%
11. \$2860 for $2\frac{1}{2}$ yr. at 6%
12. \$4750 for $1\frac{3}{4}$ yr. at 8%
13. \$5600 for $3\frac{1}{4}$ yr. at 4%

366. Interest for years and months.

1. What part of a year is 6 months? 4 months? 3 months? 2 months?

2. What is the interest on \$200 at 6% for 1 year? What part of the interest for 1 year is the interest for 6 months? for 4 months? for 3 months? for 2 months?

Then what is the interest on \$200 at 6% for 6 months? for 4 months? for 3 months? for 2 months?

3. At 5% what is the interest on \$300 for 1 year? for 4 months? for 1 yr. 4 mo.?

4. Find the interest on \$400 at 4% for 1 year; for 2 yr.; for 3 mo.; for 2 yr. 3 mo.; for 3 yr. 6 mo.

5. At 3% what is the interest of \$800 for 1 yr. 4 mo.? for 2 yr. 6 mo.? for 3 yr. 3 mo.?

In computing interest it is usual to regard a year as 12 *months, and a month as* 30 *days.*

EXERCISES

367. Find the interest for 6 months on:

1. \$100 at 4%	4. \$300 at 5%	7. \$1000 at 5%
2. \$400 at 7%	5. \$800 at 3%	8. \$3000 at 2%
3. \$500 at 3%	6. \$700 at 6%	9. \$5000 at 4%

Find the interest for 4 months on:

10. \$200 at 3%	13. \$50 at 3%	16. \$150 at 3%
11. \$600 at 2%	14. \$60 at 5%	17. \$240 at 2%
12. \$400 at 6%	15. \$90 at 4%	18. \$120 at 5%

Find the interest for three months on:

19. \$300 at 4%	22. \$110 at 8%	25. \$2000 at 5%
20. \$500 at 8%	23. \$160 at 2%	26. \$3000 at 4%
21. \$600 at 2%	24. \$250 at 4%	27. \$2400 at 2%

Find the interest for 2 months on:

28. \$300 at 2%	**31.** \$60 at 5%	**34.** \$2000 at 3%
29. \$500 at 6%	**32.** \$80 at 3%	**35.** \$1200 at 2%
30. \$400 at 3%	**33.** \$90 at 4%	**36.** \$4000 at 9%

WRITTEN EXERCISES

368. **1.** What is the amount of \$284.60 on interest for 1 yr. 8 mo. at 5%?

SOLUTION

	\$284.60,	principal	
	.05,	rate	
	\$14.2300,	int. for 1 yr.	
½ of int. for 1 yr. =	7.115,	int. for	6 mo.
⅓ of int. for 6 mo. =	2.371,	int. for	2 mo.
	\$ 23.72,	int. for 1 yr. 8 mo.	
	284.60,	principal	
	\$308.32,	amount	

Find the interest and the amount of:

2. \$350 on interest for 1 yr. 6 mo. at 6%.

3. \$475 on interest for 2 yr. 3 mo. at 4%.

4. \$3000 on interest for 4 yr. 5 mo. at 3%.

5. \$6400 on interest for 3 yr. 4 mo. at 7%.

6. \$46.50 on interest for 2 yr. 2 mo. at 8%.

7. \$85.25 on interest for 3 yr. 7 mo. at 5%.

8. \$10,000 on interest for 1 yr. 9 mo. at $3\frac{1}{2}$%.

9. \$42,000 on interest for 2 yr. 8 mo. at $2\frac{1}{4}$%.

10. \$585.75 on interest for 1 yr. 6 mo. at 6%.

11. \$648.50 on interest for 3 yr. 5 mo. at 4%.

12. \$350.80 on interest from Jan. 1, 1905, to Nov. 1, 1906, at 5%.

13. \$540.96 on interest from Apr. 9, 1904, to Mar. 9, 1906, at 6%.

369. Interest for years, months, and days.

WRITTEN EXERCISES

1. Find the amount of $524.36 on interest for 2 yr. 5 mo. 18 da. at 7%.

SOLUTION

$524.36
.07
$36.7052, int. for 1 yr.
2
$73.4104, int. for 2 yr.

⅓ of int. for 1 yr.	=	12.2350, int. for	4 mo.
¼ of int. for 4 mo.	=	3.0587, int. for	1 mo.
½ of int. for 1 mo.	=	1.5293, int. for	15 da.
⅕ of int. for 15 da.	=	.3058, int. for	3 da.
		$ 90.5392, int. for	2 yr. 5 mo. 18 da.

524.36
$614.8992

Hence, the interest is $90.54, and the amount is $614.90.

Find the interest and the amount of:

2. $60.24 for 2 yr. 4 mo. 15 da. at 5%.
3. $45.48 for 3 yr. 5 mo. 10 da. at 6%.
4. $2800 for 2 yr. 6 mo. 15 da. at 4%.
5. $5000 for 3 yr. 8 mo. 11 da. at 6%.
6. $720.16 for 5 yr. 7 mo. 18 da. at 5%.
7. $468.36 for 3 yr. 9 mo. 10 da. at 4%.
8. $25,000 for 2 yr. 5 mo. 12 da. at 6%.
9. $68,400 for 1 yr. 1 mo. 21 da. at 7%.
10. $1436.80 for 4 yr. 2 mo. 10 da. at 5%.
11. $297.45 for 2 yr. 3 mo. 15 da. at 6%.
12. $634.75 for 3 yr. 6 mo. 9 da. at 8%.
13. $1500 on interest from Sept. 7, 1906, to Dec. 31, 1908, at 4%.

REVIEW PROBLEMS IN INDUSTRIES

370. **1.** Find the number of acres in a field 30 rods by 24 rods.

2. The owner of the field used it for raising tomatoes, setting the plants in 120 rows of 99 plants each. How many plants did he set out? how many on each of the $4\frac{1}{2}$ acres?

3. When the tomatoes were picked, the yield was found to be 1125 bushels. Find the yield per row.

4. The pickers used $\frac{5}{8}$-bushel baskets and received $2\frac{1}{2}\,¢$ for each basketful picked. How much did it cost to have the entire crop picked?

5. If a bushel of tomatoes weighs 56 pounds, how many tons were taken from each acre?

6. The tomatoes were canned by the grower. If it took a ton of tomatoes to fill 384 cans, how many cans did $31\frac{1}{2}$ T. fill?

7. The 12,096 cans were packed in cases holding 24 cans each. How many cases were used? How many cases of tomatoes were produced per acre?

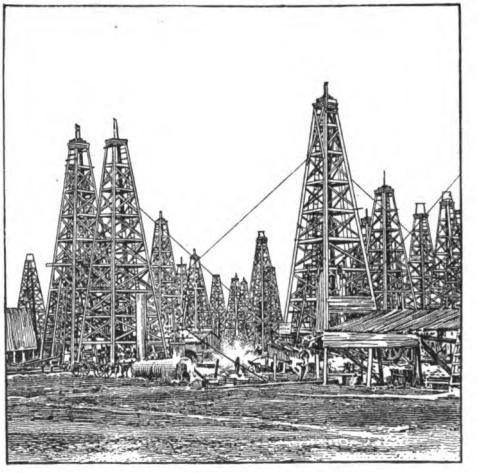
8. Find these expenses of preparing the canned tomatoes for market: 12,096 cans @ $2\,¢$; 504 cases @ $10\,¢$; putting up 504 cases of tomatoes @ $15\,¢$.

9. In addition to \$412.92, the cost of picking and canning, the expense to the producer for raising this crop was \$37.50 per acre. What was his total expense?

10. A wholesale dealer in a near-by city bought the entire crop, 12,096 cans, at $75\,¢$ per dozen cans. How much did the producer receive? What was his gain, his total expense being \$581.67?

11. If the 1125 bushels of tomatoes had not been canned but had been sold in $\frac{3}{4}$-bushel crates, the total expense would have been \$213.75. Find the difference between the gain by selling at $22\,¢$ per crate and the actual gain of \$174.33.

12. In the construction of an oil derrick in Pennsylvania, 9500 feet of lumber were used. Find the cost of the lumber at \$10.50 per M.



13. If \$99.75, the cost of the lumber, was $37\frac{1}{2}\%$ of the total cost of the derrick, what was the expense of erecting the derrick?

14. The oil well underneath was drilled to a depth of 1280 feet. Find the cost of drilling it at 55¢ per foot.

15. If the rent of the drilling machinery used was 12¢ for each foot of the well drilled, for how much did the machinery rent?

16. The first 250 feet of the well was lined with a large size of pipe called casing, as a support to the sides of the hole. How much did the casing cost at 45¢ per foot?

17. Inside the casing and extending to the bottom of the well was 1280 feet of tubing worth $15\frac{1}{2}$¢ per foot. Find its cost.

18. Besides the tubing there was the same length of pumping rods to be used in pumping the oil. Find the cost of the pumping rods at $5\frac{1}{2}$¢ per foot.

19. To make the production of oil larger, a torpedo that weighed 85 pounds was exploded at the bottom of the well. At 96¢ per pound, how much did the torpedo cost?

20. Find the combined cost of derrick and well, if the only additional expense was for well fixtures, $24.25.

21. The well in 3 weeks produced 2352 barrels of oil. Find the yield per day; per hour.

22. A barrel of crude oil is 42 gallons. Find the yield per day in gallons.

23. The operator paid rent for the land in oil instead of in money. After paying $12\frac{1}{2}\%$ of the daily yield of 112 barrels for rent, how many had he left?

24. If the price of crude oil in July was $1.35 per barrel, find the value, for that month, of the producer's portion, 98 barrels per day.

25. Find the value of the total yield during July.

26. This oil was pumped to a distant city through a pipe, interrupted at intervals by pumping stations. How far was the oil pumped, if the distances between stations were: 23.41 mi., 25.92 mi., 25.73 mi., 45.45 mi., 51.94 mi., 62.50 mi.?

27. If the yield of the well was 112 barrels per day, in how long a time did it produce enough oil to fill a tank at the end of the pipe line having a capacity of 30,800 barrels?

28. At the refinery the crude oil in a tank of this size yielded the following products, in 42-gallon barrels:

Gasoline, benzine, and naphtha,	3850 bbl.
Illuminating oil,	23,100 bbl.
Lubricating oil,	924 bbl.
Residuum and loss,	2926 bbl.

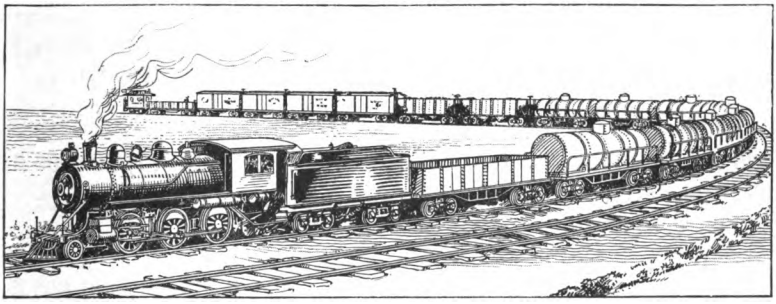
What per cent of the crude oil was each of the products?

29. Find the value of the naphtha, benzine, and gasoline at an average price of $7\frac{1}{2}¢$ per gallon.

30. The refined illuminating oil was put into 50-gallon barrels. How many such barrels were required?

31. Find the price per gallon of the illuminating oil, if it sold for $84,892.50.

32. Oil is often transported and sold in bulk instead of in barrels. When thus sold, $6\frac{1}{2}$ pounds are counted one gallon. How many gallons are there in a tank freight car that contains $24\frac{3}{8}$ tons of oil?



33. How many such tank cars, holding 7500 gallons each, would be required to load an ocean tank steamer whose capacity is 1,200,000 gallons of oil?

34. Find the weight in tons of this steamer's cargo of oil.

35. Refined oil is often shipped in 5-gallon cans, which are packed by twos in wooden cases $20\frac{3}{4}$ in. long and $10\frac{1}{2}$ in. wide. How many of these cases can be put in one layer in a compartment $41\frac{1}{2}$ ft. long and $31\frac{1}{2}$ ft. wide?

36. How many gallons will these 864 cases hold?

37. The cases are 15 in. high and there are 8640 gallons in one layer. How many gallons of oil will there be, if the cases are laid to a height of $6\frac{1}{4}$ ft.?

38. An oil delivery wagon weighed 2195 lb. empty, and 6420 lb. when filled with oil. Find its capacity in gallons.

39. The United States contributed 80,830,000 bbl. of oil to the world's production of 177,000,000 bbl. in a recent year. What per cent, to the nearest tenth, was produced by the U. S.?

When crops are sold off a farm year after year, the soil becomes impoverished. As farm animals are good agents for restoring fertility to the soil, it is often profitable when a corn crop is large and low in price, to feed the surplus crop, producing "corn-fed" beef and pork.

Hogs are often fattened along with cattle, as they eat what the cattle waste.

An Illinois farmer bought 200 steers and 50 hogs on Nov. 1, and kept the following record of feed and weight of cattle:

	NOVEMBER	
120 A. cornstalks @ $1.05	$126	Weight, Nov. 1, 190,000 lb.
25 A. pasture (75% eaten)	54	
200 bu. corn @ 24¢	48	
	DECEMBER	
Rest of pasture grass	$18	Weight, Dec. 1, 201,600 lb.
30 A. meadow grass @ $1		
1500 bu. corn @ 24¢		
¼ bbl. salt @ $1.20		
	JANUARY	
2200 bu. corn @ 27¢		Weight, Jan. 1, 213,600 lb.
20 tons hay @ $6.75		
6 tons straw @ $1.90		
¾ bbl. salt @ $1.20		
	FEBRUARY	
36 loads elevator corn @ $3		Weight, Feb. 1, 226,400 lb.
600 bu. corn @ 32¢		
18 tons hay @ $7.50		
8 tons straw @ $2		
1 bbl. salt @ $1.20		
	MARCH	
2500 bu. corn @ 35¢		Weight, Mar. 1, 240,800 lb.
16 tons hay @ $8.25		
6 tons straw @ $2		
1 bbl. salt @ $1.20		Weight, Apr. 1, 256,400 lb.

Find from the farmer's record (costs to the nearest cent):

40. Cost of cattle on Nov. 1 @ $4.10 (per 100 lb.).

41. Average cost per head.

42. Average weight per head on Nov. 1.

43. Gain in weight per head for each month.

44. Cost of feeding 1 steer during each month.

45. Cost of producing 100 lb. of beef each month.

46. Total cost of feed for five months.

47. Total gain in weight during five months.

48. Average cost of producing 100 lb. of beef.

49. Average cost of feeding 1 steer for five months.

50. How much less did it cost to produce 100 lb. of beef in November than in December?

51. How much more did it cost to produce 100 lb. of beef in March than in December?

52. If every 100 pounds of beef produced was worth $5.05, how much was gained during November? during February?

53. How much was lost during January? during March?

54. The 36 loads of corn bought in February had been slightly damaged by water. How much was saved by buying this corn, if each load was worth, for feed, as much as 50 bushels of 32-cent corn?

Apr. 1, the cattle were shipped to Chicago in 12 cars, freight 10¢ per 100 lb. on 256,400 lb., and sold through an agent at $5.05, commission $10 per car load. When sold they weighed 250,800 lb. Feed and care cost $45. Find:

55. Freight.

56. Commission.

57. Net proceeds of sale.

58. Shrinkage per head during shipment.

59. Find the total amount gained on the cattle.

60. The record for the hogs was as follows:

Nov. 1, bought 50 hogs at \$5 each.

10 hogs died during the winter.

Extra feed cost \$18.

Apr. 2, sold 40 hogs, 12,500 lb. at \$4.35 per hundredweight.

Find the gain on fattening hogs.

61. Besides the gain on cattle and hogs, the value of fertility restored to the farm by fertilizing material, was:

10% of value of grain fed, 8800 bu. worth 28¢ per bushel.

\$2 for each ton of straw and of hay (74 tons).

How much did the stock-fattening experiment benefit the farm in fertility restored?

62. Find the total gain, deducting \$130 for hired labor, \$180 for the farmer's labor, and \$85 for the use of money and tools.

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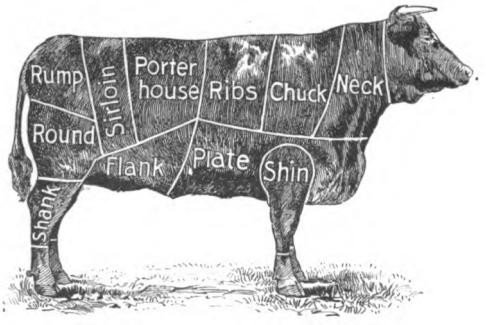
63. A steer weighing 1350 pounds yielded 783 pounds of dressed beef. What per cent of his weight was beef?



64. A packer buys this steer at \$5 per 100 pounds. Expenses amount to \$7.50. If he sells the beef at \$9.50 per 100 pounds, the hide for \$8.25, the fat for \$5.50, and the offal for \$2.25, how much will he gain?

65. Find the value of the beef at retail as follows:

Neck, 26 lb. @ 5¢; chuck, 137 lb. @ 8¢; ribs, 73 lb. @ 14¢; sirloin, 39 lb. @ 20¢; porterhouse, 94 lb. @ 22¢; rumps, 39 lb. @ 10¢; rounds, 122 lb. @ 14¢; shanks, 26 lb. @ 4¢; flanks, 47 lb. @ 6¢; plates, 120 lb. @ 5¢; shins, 60 lb. @ 4¢.

66. By the shortest postal routes San Francisco is 3250 miles west of New York City, and London is 3740 miles east. How far must a letter travel to go from San Francisco to London?

67. The first refrigerator car with southern strawberries arrived in New York in March, 1889. How long ago was that?

68. The "Congressional Limited" left Jersey City at 3:54 P.M. and arrived in Washington 4 hr. 46 min. later. At what time did it arrive in Washington?

The distance is 227 miles. Find the average speed per hour, to the nearest hundredth of a mile.

69. If 16 firemen shovel 35 tons of coal on one trip of a steamboat, how many tons will 1 fireman shovel in a season of 290 trips?

70. Mrs. Wilson bought a crate of raspberries at $3.25, and obtained from them 3 dozen pint jars of preserved fruit. She used 12 pounds of sugar costing 6¢ per pound, 3 dozen rubbers @ 8¢, and 200 cubic feet of gas for cooking at $1 per 1000 cubic feet. Making no allowance for her labor, how much did the fruit cost per jar?

71. At a clambake 2750 clams were used. Find their cost at 80¢ per 100.

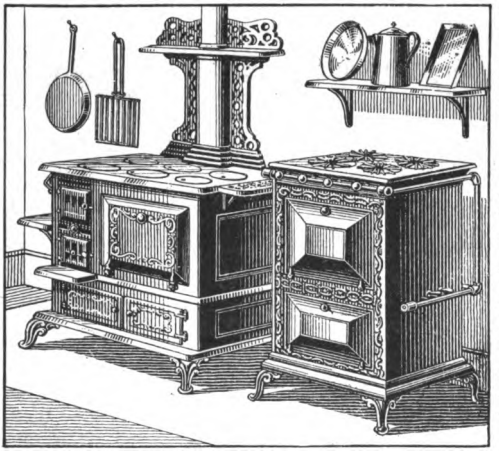
72. The circulation of a magazine is 125,000 copies per month, but only $\frac{3}{5}$ of each issue is sent by mail. Each copy weighs 1 lb. 3 oz. Find the cost of postage per month at 1¢ per pound.

73. When coal at the mine costs $3.50 per long ton, freight $.75 per short ton, and cartage and delivery $1.25 per short ton, what per cent of gain is made by a dealer who sells coal at retail at $6.50 per ton?

74. A banker purchased at the Assay Office four bars of gold, weighing in all 198 lb. 10 oz., at $20.67183 per ounce for the gold, and 40 cents per $1000 additional for the advantage of having the gold in bars. How much did the gold cost him?

75. Mr. Allen has a coal range which uses $2\frac{1}{4}$ tons of coal, costing \$5.60 per ton, during the summer. How much would that amount of coal cost him?

76. He also has a gas range. How many thousand cubic feet of gas at \$1.25 per 1000 cubic feet could he buy for \$12.60, the cost of the coal?



77. A coal range is very wasteful of the heat obtained from the coal. Often 35% of the heat passes up the flue and only 15 % is utilized in cooking, the rest going to heat the house.

According to this supposition, how many dollars' worth of coal would be utilized in cooking, and how many dollars' worth would be wasted through the flue and in heating the house unnecessarily?

78. If a coal range utilizes 15% of the heat of the coal in cooking, and a gas range 90% of the heat of the gas, and if 15,000 cu. ft. of gas gives as much heat as a ton of coal, how much gas would do Mr. Allen's cooking during the summer?

SUGGESTION. — Gas utilized = 15% of $2\frac{1}{4}$ × 15,000 cu. ft. = 90 % of gas needed.

79. How much can he save in the summer months by using gas (5625 cu. ft.) instead of coal for cooking purposes?

80. During the winter months he burns $3\frac{1}{4}$ tons of coal in the range. Of this, 15% is utilized in cooking, and 50% in heating the house. Find the value of the coal that is utilized.

81. How much does he save by using a coal range in the winter if 33,000 cu. ft. of gas do the same service as the coal?

82. If a gas retort produces 660 cu. ft. of gas from 150 lb. of coal, how much gas is obtained per long ton?

83. A gas company bought coal at \$4 per long ton and sold gas at \$1 per 1000 cu. ft. Suppose that each ton of coal produced 10,000 cu. ft. of gas, 1000 lb. of coke worth 20¢ per bushel of 40 lb., $9\frac{1}{2}$ gal. of tar worth \$1.89 per barrel of 56 gal., and ammonia liquor worth \$7.68. How many per cent was the value of the coal increased in the process of manufacture into gas and its by-products?

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84. Four men shared equally the cost of drilling a gas well for their common use. Drilling cost \$1.10 per foot and gas was obtained at a depth of 1120 feet. In addition, each man expended \$100 for piping to his house and for fixtures. How much money did each man invest?

85. In former years the average cost of fuel and light at each house was as follows, per year:

18 cords of 4-foot wood for furnace @ \$3; 6 cords of stove wood @ \$5; kerosene, \$5; taking away ashes, \$1.

Find the former annual expense to each man for heating and lighting his house.

86. After the well was dug, 93,750 cu. ft. of gas per year supplied fuel and light for each house. Since this saved the expenditure of \$90 a year, how much per thousand cubic feet was the gas worth to each man?

87. In what time did the well pay all expenses?

88. How much did each man gain on his investment of \$408, if the well held out for 10 years?

89. In a cheese factory there were 6 vats for milk. Two of them held 6250 pounds of milk each, and the other 4 held 5400 pounds each. Find the capacity of all.

90. Milk brought to this factory was paid for according to the amount of butter fat it contained. How much was paid to a farmer who brought 640 pounds of milk that tested 3.75 % butter fat, if the factory price of butter fat was 27¢ per pound?

91. A farmer, by feeding his cows $2\frac{1}{2}$ dollars' worth of meal per day more than he had been feeding them, increased the percentage of butter fat from $3\frac{1}{4}$ % to 4 % and increased the daily yield of milk from 2000 pounds to 2200 pounds. How much did he gain per day by the experiment, if he was paid 26¢ per pound for butter fat?

92. A factory produced 320,000 pounds of cheese and 28,000 pounds of butter in a year. Find the value of the output at 15¢ per pound for cheese and 25¢ per pound for butter.

93. The Mohawk Valley Coöperative Cheese Company received 237,600 pounds of milk one month, and from it produced six hundred 40-pound cheeses. How many pounds of milk were required to make 1 pound of cheese? If the rest was whey, how many pounds of whey were produced?

94. The cheese was sold at $13\frac{1}{4}$¢ per pound; freight, commission, and other charges amounted to $580; the cost of manufacture was $238.85. Find the net proceeds, or the amount divided among the farmers at the end of the month.

95. The milk received that month contained 8910 pounds of butter fat, for which $2361.15 was paid. How much then was paid the farmers for each pound of butter fat?

96. Mr. Andrews took 8400 pounds of milk to the factory that month and was credited with 308 pounds of butter fat @ $26\frac{1}{2}$¢. How much did he receive as his share of the profits? What per cent of his milk was butter fat?

97. At one time the city of Nashville was poorly lighted, but paid $47,302.75 per year for light from 382 arc lamps. How much was that per lamp, to the nearest cent?

98. After erecting its own electric light plant the city was lighted with the equivalent of 822 arc lamps at a cost of $35,162.96, including improvements, repairs, interest, etc. How much less did 822 lights cost than 382 lights formerly cost?

99. How much less did each light cost than formerly?

100. How many per cent better, to the nearest tenth of 1%, was the city lighted than before?

101. In digging deep mines it is found that the temperature of the earth near the surface is about 40° F., and that it increases 1° for every 50 feet of descent. At this rate, at what depth will the temperature be that of boiling water, 212° F.?

102. At what depth will the temperature be that of melting steel, about 2500° F.?

103. At what depth will the temperature be as high as in the arc of an arc lamp, about 5400° F.?

104. A sleeping and parlor car company charges the railroads 1¢ per car per mile for the use of its cars. Find the rent charged for twelve of these cars between Kansas City and Chicago, a distance of 458 miles.

105. A man secured a lease of a tenement for five years at $2800 a year, agreeing to make all inside repairs. He sublet the premises to 12 families at the following rates per month: $40, $35, $33, $30, $28, $25, $22, $20, $18, $15, $14, $12. Repairs cost $1425, coal $494. He lost $536 from unpaid rents and idle apartments. Find his gain.

106. A man rented a house at $492 per year, lived in it 8 mo., and sublet it for the rest of the year at $35 per month. How much rent did he pay per month for the first 8 months? How much per month did he lose during the rest of the year?

107. A school rented 4 pianos at $4.50 per month each for 10 months in a year. How much was paid for the use of pianos per year?

108. I pay my landlady $10 per month for a room and $4.20 per week for board. How much do room and board cost me per year (365 days)?

109. A man rented a typewriter at $1.25 per week and kept it from the morning of June 2 to the morning of Oct. 27. Find the amount of rent he paid.

110. A man rented a farm "on shares," agreeing to give the owner $\frac{2}{5}$ of each crop. He raised 1800 bushels of oats, 2350 bushels of wheat, and 4280 bushels of corn. How much of each crop had he left after paying the rent?

111. A farmer rented 220 acres of land at $5.25 per acre, half to be paid Sept. 1 and half Feb. 1. Find the amount of each payment.

112. A wooden freight car weighs about 30,000 pounds and has a carrying capacity of 30 tons. What part of the whole weight of car and load is the load on which freight is paid? Express the result in per cent.

113. A pressed steel car in common use weighs 34,000 pounds and has a carrying capacity of 50 tons. What per cent, to the nearest tenth, of the whole weight is the paying load?

114. A large freight vessel on the Great Lakes carries 9000 tons of iron ore at a time. At 60 cents a ton, how much is earned by such a vessel in one trip?

115. Find the value of the cargo at $4.25 per ton.

116. Such a vessel uses 450 tons of coal, costing $2.50 per ton, on a ten days' trip. How many dollars' worth of coal are consumed per day in her boiler rooms?

117. Formerly ore was unloaded from vessels by hand labor. If a gang of 25 ore shovelers could unload 1125 tons of ore in

a day and received 14 cents a ton for the job, how much did each man earn per day?

118. Recently at Conneaut, Ohio, an unloading machine operating a "clam-shell" bucket unloaded 5300 tons of ore from the steamer *James H. Hoyt* in 4 hr. 43 min. How many tons, to the nearest tenth of a ton, were unloaded per minute? how many per hour?

119. Find the cost of unloading 6000 tons of ore by hand at 14¢ per ton; by machinery at 1.4¢ per ton.

120. The bunkers of the *Connecticut* will hold 2200 tons of coal. How long will it take to fill them at the Key West coaling station by means of 4 buckets, each holding 1 ton, running on a bridge tramway, if each bucket makes a round trip in 50 seconds?

121. In one month the Bessemer and Lake Erie Railroad hauled 646,063 tons of iron ore from the Carnegie docks at Conneaut, Ohio. How many train loads of 1776 tons each were there and how many tons over?

122. How many cars holding 48 tons of ore each are there in a train, if the whole amount of ore is 1776 tons?

123. If a car of 50 (short) tons' capacity is loaded with 48 tons of iron ore, how many pounds will it be loaded beyond its regular carrying capacity?

124. A bucket worked by machinery will unload 10 tons of ore at a time from a ship. A ton of ore occupies about 17 cubic feet of space. If the unloading machine is used 8 hours per day and takes two loads per minute, how many cubic feet of ore will it transfer in 6 days?

125. A coal-loading machine transferred 12 car loads of coal, 50 tons to the car, into the hold of a vessel at Fairport, Ohio, in 45 minutes. How many tons of coal were transferred per hour?

TABLES OF DENOMINATE NUMBERS

Measures of Length

12 inches = 1 foot
3 feet = 1 yard
16½ feet = 1 rod
320 rods = 1 mile

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

Measures of Surface

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square rod
160 square rods = 1 acre
640 acres = 1 square mile

100 square feet of *roofing* is called a **square.**
A square mile of *land* is often called a **section.**

Measures of Volume

1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard

A pile of *wood* 8 ft. long, 4 ft. wide, and 4 ft. high is called a **cord.**

Liquid Measures

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

A gallon is equal to 231 cubic inches.
A gallon of water weighs about 8⅓ pounds, and a cubic foot of water about 62½ pounds or 1000 ounces.

Dry Measures

2 pints = 1 quart
8 quarts = 1 peck
4 pecks = 1 bushel

A bushel is equal to 2150.42 cubic inches, or approximately 1¼ cubic feet.

Avoirdupois Weight

16 ounces = 1 pound
100 pounds = 1 hundredweight
2000 pounds = 20 hundredweight = 1 ton

A **long ton** contains 2240 pounds. An avoirdupois pound contains 7000 grains.

A barrel of flour weighs 196 pounds.

Troy Weight

24 grains = 1 pennyweight
20 pennyweights = 1 ounce
12 ounces = 1 pound

A troy pound contains 5760 grains.

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year
366 days = 1 leap year
10 years = 1 decade
100 years = 1 century

Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four.

Centennial years exactly divisible by 400 and other years exactly divisible by 4 are leap years.

In most business computations 30 days are regarded as a month, and 360 days or 12 months as a year.

Measures of Angles and Arcs

60 seconds = 1 minute
60 minutes = 1 degree
360 degrees = 4 right angles, or 1 circumference

90° of angle is a **right angle**; 90° of arc is a **quadrant**.

Counting Table

2 = 1 pair
20 = 1 score
12 = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross

Stationers' Measures

24 sheets = 1 quire
20 quires = 1 ream

Paper is quite generally sold by the 100, 500, and 1000 sheets; also by the pound.

United States Money

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 100 cents = 1 dollar
10 dollars = 1 eagle

The denominations mill, dime, and eagle are rarely used.

Although mills are not coined, tenths of a cent are often read as mills. Dimes are always read as cents, and eagles as dollars.

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